A Discrete Causal Reconstruction of Classical Electromagnetism

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Abstract

We develop a discrete causal formulation of classical electromagnetism based on elementary formal informational carriers, termed informatons, which propagate at the invariant speed of light. These informational carriers represent the causal propagation of electromagnetic influence and are not interpreted as material particles or quanta of quantum field theory. Each informaton carries directional indices encoding attributes of the emitting source, and macroscopic electromagnetic fields emerge from the collective contribution of these informational carriers in the continuum limit. Within this framework, the electric field is interpreted as the local flux density of electric information, while magnetic phenomena emerge from geometric and kinematic effects associated with source motion.

Starting from a small set of causal emission and propagation postulates, we reconstruct the macroscopic electromagnetic field structure generated by charged particles. Within the local continuum description developed here, the formulation recovers Maxwell's equations in differential form in vacuum and reproduces the Lorentz force law. The resulting framework preserves relativistic causality and remains empirically equivalent to classical electrodynamics while providing a conceptually and geometrically transparent discrete causal reconstruction of the Maxwell–Lorentz structure.

Keywords: Classical electromagnetism, Maxwell-Lorentz theory, Maxwell's equations, Lorentz force law, discrete causal reconstruction, informaton.

1. Introduction

Classical electromagnetism is described by the Maxwell–Lorentz theory, in which electromagnetic fields satisfy Maxwell's equations and interact with charged particles through the Lorentz force law. Fields generated by moving charges are commonly represented by retarded solutions that establish the causal relation between sources and fields and account for the finite propagation speed of electromagnetic interactions [1–2]. Although the Maxwell–Lorentz framework is experimentally well established, its standard formulation is expressed in terms of

continuous fields, raising the question of whether the same field structure can be reconstructed from a discrete causal representation.

In the present work, we develop such a reconstruction based on elementary formal informational carriers, termed *informatons*, which propagate at the invariant speed c. Informatons are not interpreted as material particles or quanta of quantum field theory, but as formal carriers representing the causal propagation of electromagnetic influence. Each informaton carries directional indices encoding attributes of the emitting source and the geometry of the emission process. No assumption is made concerning their ontological status. Macroscopic electromagnetic fields emerge from their collective contribution in the continuum limit.

Within this formulation, magnetic phenomena emerge from the geometric non-collinearity between an informaton's propagation direction and its electric index, induced by source motion. The electric field is interpreted as the local flux density of electric information, while the magnetic field reflects the propagation geometry associated with informatons emitted by moving sources.

From the informational emission, propagation, and interaction principles, Maxwell's equations in differential form and the Lorentz force law are recovered, thereby reproducing the Maxwell–Lorentz structure of classical electrodynamics. The formulation is therefore presented as a conceptual reconstruction of classical electromagnetism rather than as a modification of its empirical content. It provides a discrete causal and geometrically transparent representation of the theory.

Section 2 introduces gravitational and electric information, and Section 3 formulates the informaton emission postulate and associated directional indices. Sections 4–7 develop the geometry of informaton emission and propagation, while Sections 8–12 construct the resulting macroscopic electromagnetic fields. Section 13 recovers Maxwell's equations, and Section 14 establishes the Lorentz force law.

2. Preliminary Definitions

2.1. Point Mass, Point Charge, Spacetime, Reference Frames

In the present work, material sources are represented by idealized point particles characterized by a *rest mass* (m_0) and, where applicable, an *electric charge* (q). These will be referred to as point masses or point charges. Extended bodies may be regarded as assemblies of these elementary sources, whose collective contributions determine the resulting gravitational and electromagnetic fields.

The formulation is developed in flat Minkowski spacetime within the framework of special relativity. An *inertial reference frame* (IRF) is equipped with synchronized clocks and a Cartesian spatial coordinate system (x, y, z) , with t as the time coordinate. In such a frame, free particles follow straight worldlines and light propagates in vacuum at the invariant speed c. Although the underlying formulation is relativistic, many constructions below are expressed in terms of three-dimensional spatial quantities defined within a given IRF for conceptual clarity.

2.2. The Concept of Gravitational Information (g-Information)

According to Newton's law of universal gravitation [3], two particles with masses m_1 and m_2, separated by a distance r, exert upon one another an attractive force acting along the line joining them, with magnitude

$$F_G = G\frac{m_1 m_2}{r^2},$$

where $G = 6.674 \times 10^{-11}$ $m^3kg^{-1}s^{-2}$ is the universal gravitational constant.

Consider a particle A of mass m_1 and a particle B of mass m. The gravitational force $\mathbf{F_{GB}}$ exerted by A on B is directed toward the position of A and has magnitude

$$F_{GB} = \left(\frac{Gm_1}{r^2}\right)m.$$

The gravitational force acting on particle B depends on both the source particle A and the mass of the test particle B. However, the quantity

$$g = \frac{Gm_1}{r^2}$$

depends only on the mass and position of the source particle. In classical field theory, this source-dependent influence is represented by the gravitational field. The source-dependent content represented by this field and determining the gravitational interaction with a test particle will be referred to in the present work as *gravitational information (g-information*). The term introduces no modification of Newtonian gravitation; it serves only as a formal designation for the source-dependent attributes underlying the interaction.

2.3. The Concept of Electric Information (e-information)

Two electrically charged particles in vacuum, at rest relative to an IRF, exert electric forces upon one another. The interaction is repulsive for charges of like sign and attractive for charges of opposite sign. According to Coulomb's law [4], the magnitude of the electric force between two point charges q_1 and q_2, separated by a distance r, is given by

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2},$$

where $\varepsilon_0 = 8.854 \times 10^{-12}$ F m^{-1} is the permittivity constant of free space.

Let particle A carry charge q_1, and particle B carry charge q. The electric force $\mathbf{F_{EB}}$ exerted by A on B has magnitude

$$F_{EB} = \left(\frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{r^2}\right)|q|$$

and is directed toward particle A if the charges are of opposite sign, and away from it if the charges have the same sign.

The corresponding electric field depends only on the position, magnitude, and sign of the source charge. Since it contains the source-dependent content that determines the electric interaction with a test charge, this content will be referred to in the present work as *electric information* (*e-information*). As in the gravitational case, the term introduces no modification of

classical electrodynamics but serves as a formal designation for the source-dependent attributes underlying the interaction.

These concepts provide the basis for the discrete causal formulation developed below, in which such source-dependent information is represented by propagating informatons.

3. The Postulate of the Emission of Informatons

In order to provide a discrete and locally causal representation of the field structure associated with gravitational and electric interactions, the following postulate is introduced.

A particle is assumed to continuously emit discrete informational carriers, hereafter referred to as informatons. Collectively, these carriers encode the information associated with the emitting particle. They always transport g-information and, in the case of electrically charged particles, additional e-information.

Informatons are introduced as formal elements whose collective behavior reproduces, in an appropriate continuum limit, the classical gravitational and electric fields. They propagate away from their source at the invariant speed c and encode attributes of the emitting particle associated with mass, position, velocity, and - where applicable - electric charge. Their propagation is assumed to respect relativistic causality. The ensemble of informatons emitted by a particle constitutes a structured geometrical distribution which, at the macroscopic level, corresponds to the classical field associated with that particle.

Informatons are not directly observable and are not identified with particles of matter or with quanta of standard quantum fields. They are introduced as discrete informational carriers whose collective distribution reproduces the classical field description in the continuum limit. No assumption is made that individual informatons carry localized energy or momentum; energy, momentum and inertia remain attributes of matter and of the emergent classical fields. The informaton framework is intended as a kinematical and structural description whose validity is assessed through its ability to reproduce established interaction laws.

3.1. Rules of Informaton Emission

The emission of informatons by a particle at rest in an IRF **O** is governed by the following rules.

A. Emission Kinematics

1) Informatons are emitted isotropically in the instantaneous rest frame of the emitting particle and propagate radially outward at speed c. In other inertial reference frames, the corresponding emission pattern is obtained by Lorentz transformation, ensuring Lorentz covariance of the emission process.

2) The emission process is characterized by a constant average emission rate $\dot{N} = dN/dt$, proportional to the rest mass m_0 of the emitting particle:

$$\dot{N} = Km_0.$$

The proportionality constant

$$K = \frac{c^2}{h} = 1.36 \times 10^{50}\ \mathrm{kg^{-1}\ s^{-1}},$$

sets the scale of the informaton emission rate. With this choice, a change in rest energy $\Delta E_0 = h\nu$ - for example, associated with the absorption or emission of a photon of frequency ν - corresponds to a rest-mass variation $\Delta m_0 = h\nu/c^2$, and hence to a change in emission rate

$$\Delta \dot{N} = K\Delta m_0 = \nu.$$

Thus, the variation in emission rate is equal to the frequency of the associated energy exchange.

The *directional indices* introduced below are chosen such that the collective contribution of informatons reproduces the classical gravitational and electric field strengths in the continuum limit. Their definitions therefore establish the correspondence between the discrete informaton representation and the conventional field description.

B. Gravitational Index (g-Index)

Each informaton emitted by a particle carries a vector attribute, denoted $\mathbf{s_g}$, referred to as the *gravitational index* (*g-index*).

1) The gravitational index is radial and oriented toward the position of the emitting particle.

2) The magnitude of the gravitational index is defined as

$$s_g = \frac{1}{K\eta_0} = \frac{h}{\eta_0 c^2} = 6.18 \times 10^{-60}\ \mathrm{m^3 s^{-1}},$$

where

$$\eta_0 = \frac{1}{4\pi G} = 1.19 \times 10^{9}\ \mathrm{kg s^2 m^{-3}}.$$

C. Electric Index (e-Index)

Informatons emitted by electrically charged particles carry, in addition to the gravitational index, an attribute related to electric charge, referred to as the *electric index* (*e-index*) and denoted $\mathbf{s_e}$.

1) The electric index is radial relative to the position of the emitter. Its direction is outward for positive charges and inward for negative charges.

2) The magnitude of the electric index depends on the charge-to-mass ratio Q/m_0 of the emitting particle and is defined as

$$s_e = \frac{1}{K.\,\varepsilon_0}\frac{Q}{m_0} = 8.32 \times 10^{-40}\ \frac{Q}{m_0}\mathrm{kg m^3 C^{-1} s^{-1}},$$

where $Q = |q|$

As in the gravitational case, the electric index specifies the elementary contribution of a single informaton to the electric field representation. Its dependence on the ratio Q/m_0 ensures that, through the collective contribution of many informatons, the classical Coulomb field is recovered in the macroscopic continuum limit.

3.2. Concluding Remark

The preceding postulates define the elementary constituents of the discrete causal formulation introduced in this work. The following sections investigate the collective behavior of informatons and demonstrate how the classical electromagnetic field structure emerges from their collective contribution in the continuum limit.

4. The Electric Field of a Point Charge at Rest

4.1. The Emission of Informatons by a Particle at rest

Consider (fig 1) a charged particle of rest mass m_0 and charge q, at rest at the origin of an IRF **O**. According to the postulate of informaton emission, the particle continuously emits informatons that propagate radially outward at the invariant speed c, carrying elementary contributions to the gravitational information and, where applicable, to the electric information.

Because the emission process is stationary and isotropic in the rest frame of the particle, the total emission rate of informatons $\dot{N}$ equals the number of informatons crossing per unit time any closed surface enclosing the particle. According to section 3.1 the emission rate is given by

$$\dot{N} = \frac{dN}{dt} = Km_0$$

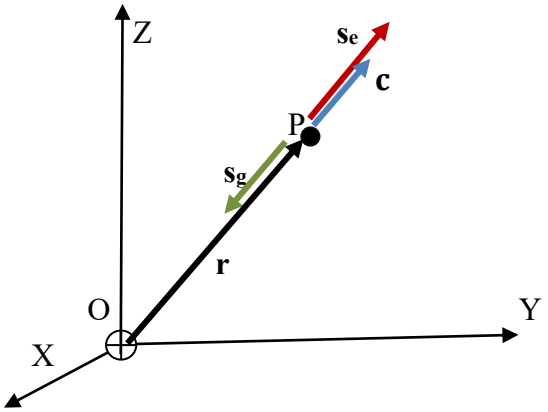


Figure 1. An informaton[1] emitted by a positively charged particle at rest in an IRF

Owing to the extremely large value of $\dot{N}$, the ensemble of informatons emitted by the particle may be treated, to excellent approximation, as a continuous informational flux. For a spherical surface of radius r centered on the particle, the flux density of informatons - defined as the number of informatons crossing per unit time and per unit area an elementary surface perpendicular to the direction of propagation - is therefore

$$\frac{\dot{N}}{4\pi r^2} = \frac{Km_0}{4\pi r^2}. \qquad (1)$$

For any closed surface that does not enclose the particle, the inward and outward informaton fluxes balance, leaving the number of informatons within the enclosed region constant. On macroscopic scales, even a small volume contains a very large number of informatons, allowing their discrete distribution to be approximated by a continuous density.

[1]For visualization purposes only, an informaton may be schematically represented as a point propagating at velocity $\boldsymbol{c}$; the associated g- and e-indices are represented by the vectors $\mathbf{s_g}$ and $\mathbf{s_e}$.

In the continuum limit, the collective distribution of informatons reproduces the classical field associated with the particle. Depending on whether gravitational or electric information is considered, this field corresponds respectively to the classical gravitational or electric field. In the present work, the focus is on electromagnetic phenomena; the particle is therefore treated as a point charge. An analogous treatment of the gravitational case, where the particle is modeled as a point mass, has been considered in earlier work [5–8].

4.2. The Electric Field (e-Field) of a Point Charge at Rest

Consider (fig 1) a point P located at a distance r from a point charge q at rest. Informatons passing through the neighborhood of P propagate radially outward with velocity $\mathbf{c} = c\,\mathbf{u_r}$, where $\mathbf{u}_r = \mathbf{r}/r$ is the radial unit vector.

According to the postulate introduced in Section 3.1, each informaton carries an electric index $\mathbf{s_e}$ given by

$$\mathbf{s}_\mathrm{e} = \frac{q}{m_0}\frac{1}{K\varepsilon_0}\,\mathbf{u_r} \tag{2}$$

The magnitude of $\mathbf{s_e}$ depends on the charge-to-mass ratio Q/m_0, and its orientation reflects the sign of the electric charge.

The electric information flux density at P is given by the product of the informaton flux density (1) and the electric index (2):

$$\frac{Km_0}{4\pi r^2}\,\frac{q}{m_0}\frac{1}{K\varepsilon_0} = \frac{q}{4\pi\varepsilon_0 r^2}.$$

This quantity, together with the radial orientation of the electric indices, defines the electric field at point P. The *electric field vector* produced by a point charge at rest is therefore

$$\mathbf{E} = \frac{\dot{N}}{4\pi r^2}\mathbf{s}_e = \frac{q}{4\pi\varepsilon_0 r^2}\,\mathbf{u_r} = \frac{q}{4\pi\varepsilon_0 r^3}\mathbf{r}$$

At any point in space, **E** points radially away from the source for $q > 0$ and toward the source for $q < 0$. Its magnitude $E = |\mathbf{E}|$ represents the rate per unit area at which electric information crosses a surface perpendicular to **E**.

4.3. The Electric Flux (e-Flux) through a Surface Element

Consider an infinitesimal oriented surface element dS at a point P, represented by

$$\mathbf{dS} = dS\mathbf{u_n},$$

where dS denotes the scalar area of the element and $\mathbf{u_n}$ is the unit normal vector at P.

Let **E** be the electric field at that point. The differential *electric flux* $d\Phi_E$ through the surface element is defined by the scalar product

$$d\Phi_E = \mathbf{E}\cdot\mathbf{dS}.$$

Equivalently,

$$d\Phi_E = E\,dS\cos\,\alpha,$$

where $E = |\mathbf{E}|$ and α is the angle between the electric field and the unit normal to the surface element. The flux is positive when the electric field crosses the surface element in the direction of the chosen unit normal, and negative when it crosses it in the opposite direction.

The differential electric flux is defined as the scalar product of the electric field with the oriented surface element. Within the present framework, this quantity is interpreted as the passage of electric information across the surface element in the direction of its chosen orientation. This interpretation introduces no additional dynamical assumptions; it reformulates the standard geometric definition of flux in terms consistent with the discrete emission picture.

4.4. The Electric Field of a Set of Point Charges at Rest

Consider a system of point charges $q_1,\ldots, q_n$ all at rest in an IRF **O**. At an arbitrary point P, each charge q_i generates an electric field $\mathbf{E_i}$. The differential electric flux, $d\Phi_E$, through an infinitesimal surface element dS at P is the sum of the individual contributions:

$$d\Phi_E = \sum_{i=1}^{n} \mathbf{E_i} \cdot \mathbf{dS} = \left(\sum_{i=1}^{n} \mathbf{E_i}\right) \cdot \mathbf{dS} = \mathbf{E} \cdot \mathbf{dS}$$

Hence, the resultant electric field at point *P* is

$$\mathbf{E} = \sum_{i=1}^{n} \mathbf{E_i}.$$

This relation expresses the *principle of superposition*: the electric field produced by a system of stationary point charges is the vector sum of the fields generated by the individual charges.

4.5. The Electric Field of a Charge Continuum at Rest

A spatially bounded object possessing a continuous, time-independent distribution of electric charge will be referred to as a *charge continuum*. The local charge distribution is characterized by the charge density ρ_E, defined as

$$\rho_E = \frac{dq}{dV},$$

where dq denotes the charge contained within the infinitesimal volume element dV.

A continuous charge distribution at rest in an IRF may be regarded as composed of infinitesimal charge elements. Each element produces an elementary contribution **dE** to the electric field at an arbitrary point P. The total electric field at P is obtained by integrating over the entire volume of the continuum.

4.6. Conservation of Electric Information and Gauss' Law

Within the present framework,

$$\Phi_E = \oiint \mathbf{E} \cdot \mathbf{dS},$$

the total electric flux through a closed surface S, represents the net outward flux of electric information across the surface. According to the definition of the electric field and the isotropic emission of informatons established in Sect. 4.1, *this flux equals the total electric charge enclosed by the surface divided by* ε_0.

For a single point charge q enclosed by S**:**

$$\oiint \mathbf{E} \cdot \mathbf{dS} = \frac{q}{\varepsilon_0}.$$

For a system of point charges $q_1, \ldots, q_n$:

$$\oiint \mathbf{E} \cdot \mathbf{dS} = \frac{1}{\varepsilon_0} \sum_{i=1}^{n} q_i.$$

For a continuous charge distribution characterized by the charge density ρ_E:

$$\oiint \mathbf{E} \cdot \mathbf{dS} = \frac{1}{\varepsilon_0} \iiint_V \rho_E dV.$$

These equivalent expressions constitute *Gauss's law* in integral form. By application of Ostrogradsky's divergence theorem [9], the last relation is locally equivalent to

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho_E}{\varepsilon_0}.$$

4.7. Conservativity of the Electrostatic Field

For stationary charge distributions, the electric field obtained by informaton superposition coincides with the Coulomb field and is therefore conservative [3]. Hence,

$$\oint \mathbf{E} \cdot \mathbf{dl} = 0 \qquad \text{or equivalently} \qquad \boldsymbol{\nabla} \times \mathbf{E} = 0.$$

Consequently, the electrostatic field can be expressed as the gradient of a scalar potential V:

$$\mathbf{E} = -\boldsymbol{\nabla} V.$$

5. The Electrostatic Interaction between Point Charges at Rest – Coulomb's Law

The preceding sections established that a stationary electric charge generates a static electric field through the stationary and isotropic emission of electric information, and that the resulting informational field coincides in the continuum limit with the classical electrostatic field. We now consider how this field influences a charged particle and show that the present formulation leads to the electric force law and the resulting acceleration of the particle. Applied to the electrostatic field of a point charge, the force law yields Coulomb's law.

5.1. The Electric Force on a Charged Particle in an Electric Field

Consider a set of stationary point charges in an IRF **O**, generating an electrostatic field **E**. Focus on a particle of charge q located at point P. Its electric self-field, which would be spherically symmetric in isolation, is disturbed by the e-information emitted by the surrounding particles. The external electric field provides the quantitative measure of this disturbance.

The central dynamical postulate of the present framework is that *a disturbance of the symmetry of a charged particle's informational self-field produced by the external informational environment gives rise to a restoring tendency whose magnitude is proportional both to the informational disturbance and to the strength of the self-field.*

Since the strength of the particle's informational self-field is proportional to the magnitude of its charge, the restoring tendency is proportional to both **E** and q, with its direction determined by their relative sign. Its observable mechanical manifestation is identified with the electric force, which is here defined by the linear constitutive relation

$$\mathbf{F_E} = q\mathbf{E}.$$

If the particle is free to move, the resulting acceleration **a** *provides the mechanical response through which the local informational imbalance is resolved.*

5.2. Motion of a Charged Particle in an Electric Field

To represent this response mechanically, we introduce a *virtual gravitational field*[2] $\bar{\mathbf{E}}_\mathbf{g}$, defined as the gravitational field that produces on a particle of rest mass m_0 and charge q the same mechanical effect as the electric field **E**.

By definition, the corresponding forces are equal

$$\mathbf{F_G} = \mathbf{F}_\mathrm{E},$$

where $\mathbf{F_G} = m_0\bar{\mathbf{E}}_\mathbf{g}$ denotes the gravitational force associated with $\bar{\mathbf{E}}_\mathbf{g}$.

It follows that

$$\bar{\mathbf{E}}_\mathbf{g} = \frac{q}{m_0}\mathbf{E}$$

By construction, the virtual gravitational field produces the same mechanical effect as the electric field. Using the local equivalence between a gravitational field and an accelerated reference frame, this effect may be represented locally by the corresponding free-fall acceleration. Hence, for a freely moving charged particle,

$$\mathbf{a} = \bar{\mathbf{E}}_\mathrm{g} = \frac{q}{m_0}\,\mathbf{E}.$$

Multiplying both sides by m_0 yields

$$\mathbf{F}_E = q\,\mathbf{E} = m_0\mathbf{a},$$

which is Newton's second law for a charged particle in an electric field in the nonrelativistic regime. Within the informational interpretation, this acceleration is the mechanical response through which the disturbance of the local symmetry of the particle's self-field is resolved.

5.3. Coulomb's Law

Consider (fig. 2) two point charges, q_1 and q_2, at rest at positions P_1 and P_2 respectively, in IRF **O**. Let **r** be the vector pointing from q_1 to q_2, with magnitude r and corresponding unit vector $\mathbf{u_{12}} = \mathbf{r}/r$.

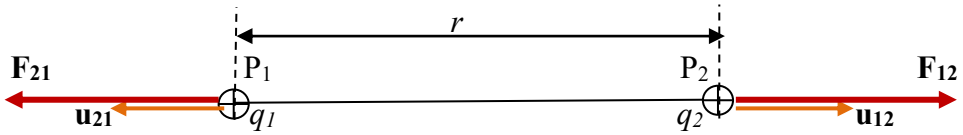


Figure 2. Coulomb's law

The charge q_1 generates at the position P_2 an electric field

$$\mathbf{E_2} = \frac{q_1}{4\pi\varepsilon_0 r^2}\,\mathbf{u_{12}}$$

The force exerted by that field on q_2 is:

[2] The introduction of the virtual gravitational field serves here as a formal device and does not imply the existence of an additional physical field.

$$\mathbf{F_{12}} = q_2\mathbf{E_2} = \frac{q_1 . q_2}{4\pi\varepsilon_0 r^2}\,\mathbf{u_{12}}$$

Similarly, the charge q_2 generates at the position of q_1 an electric field that produces a force

$$\mathbf{F_{21}} = -\mathbf{F_{12}}.$$

This result constitutes Coulomb's law for the electrostatic interaction between two point charges at rest. The forces act along the line joining the two charges, are equal in magnitude and opposite in direction, and therefore satisfy Newton's third law.

5.4. Concluding Remark

Within the present framework, the electrostatic interaction between charged particles is interpreted as the dynamical response of a charge to a disturbance of the informational field distribution in its immediate vicinity. This response admits a mechanical representation through a virtual gravitational field, thereby leading naturally to the standard electric force law

$$\mathbf{F_E} = q\mathbf{E},$$

from which Newton's second law and Coulomb's law follow. The present formulation thus provides a discrete and locally causal reconstruction of electrostatic interaction whose continuum limit reproduces the standard results of classical electrostatics.

6. The Electric Field of a Point Charge Moving with Constant Velocity

We now determine the electric field of a point charge moving with constant velocity relative to an IRF **O**.

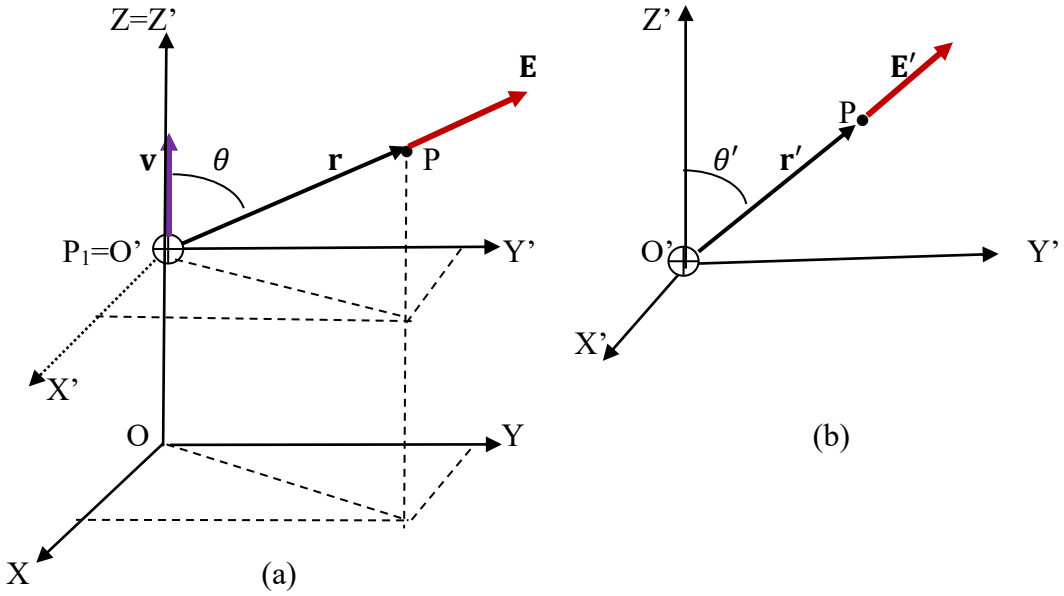


Figure 3. The e-field of a point charge moving with constant velocity

Consider a (positively) charged point particle (fig 3,a) that is moving with constant velocity $\mathbf{v} = v.\mathbf{u_z}$ along the *Z*-axis of an IRF **O**. We assume that its rest mass m_o and its charge q are independent of the velocity **v**. At the moment $t = 0$, it passes at the origin O and at the time t at the point P_1. Its position is there given by $z_{P_1} = vt$. Let P be an arbitrary observation point, and let

$$\mathbf{r} = \mathbf{r}_{\mathrm{P}} - \mathbf{r}_{\mathrm{P}_1}$$

denote its position relative to P_1.

Let **O'** denote the particle's proper IRF (fig 3,b), with its origin O' fixed at the charge and assume that $t = t' = 0$ when O' passes through O. The position of P relative to **O'** is given by **r' = O'P**. In this frame, the charge is at rest. Since the electric field of a charge at rest has already been obtained in its proper frame, the field observed in an arbitrary IRF may be determined by transforming the corresponding e-information flux densities according to the Lorentz transformation.

In **O'** the electric field $\mathbf{E}'$ at P is defined by §4.2:

$$\mathbf{E}' = \frac{q}{4\pi\varepsilon_0 r'^3}\,\mathbf{r}'.$$

Since the electric field is identified with the local e-information flux density, its Cartesian components represent the e-information flux densities normal to the corresponding elementary surface elements. In **O'**, the associated e-information fluxes are

$$E'_{x'}dy'dz' = \frac{q}{4\pi\varepsilon_0 r'^3}x'dy'dz'$$

$$E'_{y'}dz'dx' = \frac{q}{4\pi\varepsilon_0 r'^3}y'dz'dx'$$

$$E'_{z'}dx'dy' = \frac{q}{4\pi\varepsilon_0 r'^3}z'dx'dy'$$

The e-information carried by a given bundle of informatons is conserved during propagation. The transformation of these flux quantities between **O** and **O'** may therefore be obtained by applying the Lorentz transformation to the corresponding surface elements. The coordinates of P in the frames **O** and $\mathbf{O}'$ are related by the Lorentz transformation [10], with

$$x' = x, \qquad y' = y, \qquad z' = \frac{z - vt}{\sqrt{1-\beta^2}},$$

and

$$r' = r\frac{\sqrt{1-\beta^2\sin^2\theta}}{\sqrt{1-\beta^2}},$$

where $\beta = v/c$ and θ denotes the angle between the position vector **r** and the velocity **v**, measured in frame **O**.

Substituting these relations gives the electric field components in **O**:

$$E_x = \frac{q}{4\pi\varepsilon_0 r^3}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{\frac{3}{2}}}\,x,$$

$$E_y = \frac{q}{4\pi\varepsilon_0 r^3}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{\frac{3}{2}}}\,y,$$

$$E_z = \frac{q}{4\pi\varepsilon_0 r^3}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}}\,(z - z_{P_1}).$$

Consequently, the electric field can be written in vector form as

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^2}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}}\,\mathbf{u_r},$$

where $\mathbf{u_r} = \mathbf{r}/r$.

Within the present formulation, a point charge in uniform rectilinear motion relative to **O** thus generates a time-dependent electric field that remains radially directed with respect to its instantaneous position in the chosen IRF. For $q > 0$, **E** points away from the charge; for $q < 0$, it points toward it. The field magnitude is

$$E = \frac{|q|}{4\pi\varepsilon_0 r^2}\frac{1-\beta^2}{(1-\beta^2 sin^2\theta)^{3/2}}.$$

For $v \ll c$, $\beta \ll 1$ and the velocity-dependent factor approaches unity. The field therefore reduces to the Coulomb form

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^3}\,\mathbf{r}.$$

Physically, this limit corresponds to the source displacement during the propagation time becoming negligible relative to the source–observation distance.

Integration of the angular dependence over a spherical surface centered on the instantaneous position of the charge gives

$$\oiint \mathbf{E}\cdot\mathbf{dS} = \frac{q}{\varepsilon_0}$$

in accordance with Gauss's law and, within the present framework, with conservation of electric information.

The preceding derivation shows that Rule C.1 of the postulate of informaton emission represents the special case of a source at rest. The Lorentz transformation of the e-information flux leads to a more general kinematic orientation rule, whose extension to accelerating sources will be developed in Chapter 12.

7. The Magnetic Field of a Point Charge Moving with Constant Velocity

7.1. The Emission of Informatons by a Point Charge Moving with Constant Velocity

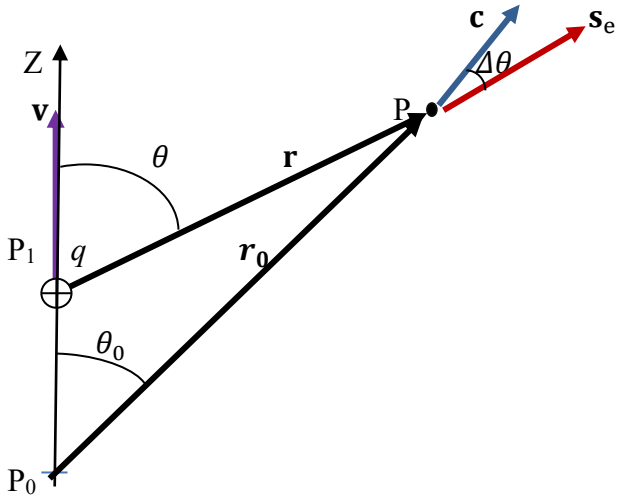


Figure 4. The emission of an informaton by a point charge moving with constant velocity

Consider (fig 4) a point charge q moving with constant velocity **v** along the *Z*-axis of an IRF **O**. At time t, the charge passes point P_1. Let P be a fixed observation point. Its position relative to the instantaneous position of the charge is given by the vector $\mathbf{r} = \mathbf{P_1P}$. Because P_1 varies with time, the vector **r**, its magnitude r, and the angle θ between **r** and **v** are time-dependent.

The informatons reaching the observation point P at time t propagate with speed c and were emitted earlier, at the corresponding *retarded time* when the particle was located at P_0. Since $P_0P = r_0$, the propagation time is $\Delta t = r_0/c$. During this interval, the emitter moves to P_1, with $P_0P_1 = v\,\Delta t$.

1) According to Rule A.1 of the emission postulate, the velocity vector **c** of an informaton is directed along its trajectory, i.e. along the radius P_0P.

2) Rule C.1 asserts, consistently with the field direction discussed in §6, that the e-index $\mathbf{s_e}$ is oriented along P_1P.

The propagation direction and the direction of the electric index therefore no longer coincide. In other words, motion of the source leads to *a non-collinearity between the propagation direction* **c** *and the electric index* $\mathbf{s}_e$ *of an informaton*. Let $\Delta\theta$ denote the angle between these vectors, defined by

$$\cos(\Delta\theta) = \frac{\mathbf{c}\cdot\mathbf{s}_e}{cs_e}.$$

This angle will be referred to as the *characteristic deviation* of the informaton.

The index $\mathbf{s}_e$ may consequently be decomposed into a component parallel to **c** and a component transverse to it. The magnitude of the transverse component is

$$s_b = s_e \sin(\Delta\theta),$$

corresponding to the projection of $\mathbf{s_e}$ orthogonal to the propagation direction.

This transverse component at P is represented by the vector

$$\mathbf{s_b} = \frac{\mathbf{c}\times\mathbf{s}_e}{c}.$$

Because this transverse contribution is on the macroscopic level associated with what is described as the *magnetic aspect* of the generated field, we call it the *magnetic index* (*b-index*) of the informaton.

The b-index is perpendicular to the plane defined by the propagation path and the emitter trajectory, with orientation determined by the right-hand rule and magnitude $s_b = s_e \sin(\Delta\theta)$. Its orientation is reversed for a negative charge.

7.2. The Magnetic Field Vector – General Definition

Each informaton emitted by a moving point charge and reaching an observation point carries, in addition to its e-index , the kinematically defined b-index

$$\mathbf{s_b} = \frac{\mathbf{c}\times\mathbf{s_e}}{c} = \mathbf{u_c}\times\mathbf{s_e},$$

where $\mathbf{u_c}$ is the unit vector along the propagation direction of the informaton.

Within the informational formulation, this index represents the magnetic information carried by the informaton. Let n denote the local *informaton volume density*, defined as the number of informatons per unit volume. The corresponding b-information density defines the *magnetic field vector*

$$\mathbf{B} = n\,\mathbf{s_b} = n\frac{\mathbf{c}\times\mathbf{s_e}}{c}$$

Since the informatons propagate at speed c, the local informaton flux density and the volume density are related by $N=cn$. Together with $\mathbf{E}=N\,\mathbf{s_e}$, this definition relates the magnetic field directly to the electric field and the local propagation geometry.

$$\mathbf{B} = \frac{\mathbf{c}\times\mathbf{E}}{c^2}$$

A moving point charge therefore generates a unified *electromagnetic field*, understood here as a dual informational structure consisting of

1) an electric field **E**, representing the local e-information flux density, and

2) a magnetic field **B**, representing the local density of b-information.

Within the present informational framework, these two fields constitute complementary manifestations of the same underlying process, namely the emission and propagation of informatons by a moving charged particle.

7.3. The Magnetic Field Vector of a Point Charge Moving with Constant Velocity

Applying the law of sines to the triangle P_0P_1P (refer to fig. 4), we obtain:

$$\frac{\sin(\Delta\theta)}{v\,\Delta t} = \frac{\sin\theta}{c\,\Delta t}$$

from which it follows that

$$s_b = s_e\,\frac{v}{c}\sin\,\theta.$$

Taking into account the orientation of the vectors involved, the b-index of an informaton emitted by a point charge moving with constant velocity **v** can thus be written as

$$\mathbf{s_b} = \frac{\mathbf{v}\times\mathbf{s_e}}{c}.$$

From the general definition of the magnetic field vector in §7.2, it follows that, for a point charge moving uniformly with velocity **v**, the magnetic field at point P is given by

$$\mathbf{B} = \frac{\mathbf{v}}{c^2}\times(N\,\mathbf{s_e}) = \frac{\mathbf{v}\times\mathbf{E}}{c^2}.$$

This relation shows that the magnetic field is not introduced as an independent entity but emerges directly from the electric information carried by moving informatons.

Taking into account the expression for the electric field derived in §6, the magnetic field vector produced at P by a charge q moving with constant velocity **v** is

$$\mathbf{B} = \frac{\mu_0 q}{4\pi r^3}\,\frac{(1-\beta^2)}{(1-\beta^2 sin^{\,2}\,\theta)^{\frac{3}{2}}}\,(\mathbf{v}\times\mathbf{r}),$$

where

$$\mu_0 = \frac{1}{\varepsilon_0 c^2} = 1.26 \times 10^{-6}\ \mathrm{H\ m^{-1}}$$

is the permeability constant.

A point charge moving with constant velocity relative to an IRF thus generates a magnetic field whose magnitude is

$$B = \frac{\mu_0 \mid q \mid}{4\pi r^2} \frac{(1-\beta^2)}{(1-\beta^2 \sin^2\theta)^{3/2}} v \sin\theta,$$

and whose direction is determined by the vector product $(\mathbf{v} \times \mathbf{r})$. For a positive charge, **B** is parallel to $(\mathbf{v} \times \mathbf{r})$, and for a negative charge it is antiparallel.

In the non-relativistic limit $v \ll c$, this reduces to

$$\mathbf{B} = \frac{\mu_0 q}{4\pi r^3}(\mathbf{v} \times \mathbf{r}).$$

In this regime, the displacement of the source during the informaton propagation time is small compared with the source–observation distance, so that the characteristic deviation remains small. Consequently $s_b = s_e \sin(\Delta\theta)$, may be evaluated to first order in *v/c*, yielding the standard non-relativistic magnetic field.

Within the present framework, no additional fundamental postulate is required for the magnetic field: magnetism emerges kinematically from the non-collinearity between the propagation direction and the electric index of informatons emitted by a moving charge.

8. The Electromagnetic Field of a Point Charge Moving with Constant Velocity

The electric and magnetic fields derived in Sections 6 and 7 constitute the electromagnetic field associated with a uniformly moving point charge. The field is completely specified by the vector pair (**E**, **B**), whose components arise from the e-information and b-information carried by the emitted informatons. Both field components are therefore fully determined by the charge, its velocity, and the geometrical relation between the source and the observation point.

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^3} \frac{(1-\beta^2)}{(1-\beta^2 sin^2\theta)^{3/2}} \mathbf{r}$$

$$\mathbf{B} = \frac{\mu_0 q}{4\pi r^3} \frac{(1-\beta^2)}{(1-\beta^2 sin^2\theta)^{3/2}} (\mathbf{v} \times \mathbf{r})$$

In the non-relativistic limit ($v \ll c$), these expressions reduce to the familiar Coulomb field and the magnetostatic field of a slowly moving point charge.

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^3} \mathbf{r} \qquad \mathbf{B} = \frac{\mu_0 q}{4\pi r^3}(\mathbf{v} \times \mathbf{r})$$

9. The Electromagnetic Field of a Set of Moving Point Charges

Consider a system of point charges $q_1, q_2, \dots, q_n$ that, at a given instant t, move with velocities $\mathbf{v_1}, \mathbf{v_2}, \dots, \mathbf{v_n}$ relative to an IRF $\mathbf{O}$. The ensemble of informatons emitted by all charges defines an electromagnetic field that, in the IRF $\mathbf{O}$, is characterized at each point P by the vector pair $(\mathbf{E}, \mathbf{B})$.

1) Each charge contributes an electric field component $\mathbf{E_i}$ to the total field at P. According to the superposition principle (§4.4), the effective electric field vector at P is:

$$\mathbf{E} = \sum \mathbf{E_i}.$$

2) Because the charges are in motion, each contributes a magnetic field component $\mathbf{B_i}$ at point P. Since informatons propagate independently, their contributions combine linearly. Thus, the total magnetic field at P is given by

$$\mathbf{B} = \sum \mathbf{B_i}.$$

The electromagnetic field generated by a system of moving point charges is therefore obtained by the linear superposition of the individual field contributions carried by the emitted informatons. This result provides the basis for the treatment of current and continuous charge distributions considered in the following sections.

10. The Electromagnetic Field of a Current-Carrying Conductor

A *current-carrying conductor* realizes a steady transport of electric charge confined within a material structure.

Consider an arbitrary cross-section ΔS of the conductor, perpendicular to the direction of charge transport. If an amount of charge dq passes through this section during an infinitesimal time interval dt, the rate of charge transport - measured with respect to a chosen reference direction - is

$$i = \frac{dq}{dt}.$$

The sign of i depends both on the sign of the transported charge and on the chosen reference direction along the conductor. The scalar quantity i is called the *electric current* through the conductor.

In a cylindrical conductor, the charge carriers drift along the axis with a velocity $\mathbf{v}$ which is typically extremely small compared with the speed of light. For a steady current, one may idealize the conductor as a one-dimensional filament along which the current flows. If during a time interval dt the carriers traverse a segment $\mathbf{dl} = \mathbf{v}\,dt$, the transported charge is $dq = i\,dt$. It follows that

$$\mathbf{v}\,dq = i\,\mathbf{dl}.$$

The quantity $i\,\mathbf{dl}$ defines the *current element*.

In a conductor that is electrically neutral as a whole, the charge density of the mobile carriers is balanced by an oppositely charged stationary background. To an excellent approximation, their electrostatic fields cancel in the exterior region, whereas the magnetic contributions associated

with charge motion do not. In informational terms, the corresponding e-information contributions cancel, while the motion-dependent b-information does not. The conductor therefore generates a magnetic field without producing an observable external electric field in the laboratory frame.

The magnetic field **dB** produced at a point P by a current element i **dl** may be obtained from the expression derived in §7.3 for a moving point charge, replacing the factor $\mathbf{v}\,dq$ by i **dl** one obtains

$$\mathbf{dB} = \frac{\mu_0 i}{4\pi r^3}\ \frac{1-\beta^2}{(1-\beta^2 \sin^2\theta)^{3/2}}\ (\mathbf{dl}\times\mathbf{r}),$$

where **r** is the vector from the current element to the observation point and θ is the angle between **dl** and **r**.

Since the drift velocity in ordinary conductors satisfies $v \ll c$, relativistic corrections are negligible; for a steady current, the macroscopic source distribution is time-independent. The expression therefore reduces to

$$\mathbf{dB} = \frac{\mu_0 i}{4\pi r^3}\ (\mathbf{dl}\times\mathbf{r}),$$

which is the classical *Biot–Savart law*.

For a long, straight wire carrying a steady current i, integration of the Biot–Savart law yields a magnetic field with magnitude

$$B = \frac{\mu_0\, i}{2\pi r}$$

at a distance r from the wire. The direction of **B** is related to the sense of the current by the right-hand (corkscrew) rule.

For a long straight wire, integration around a circular path enclosing the wire gives

$$\oint \mathbf{B}\cdot\mathbf{dl} = \mu_0 i$$

More generally, for steady currents this relation takes the magnetostatic form

$$\oint \mathbf{B}\cdot\mathbf{dl} = \mu_0 \sum i_{in}$$

which is *Ampère's law*.

11. The Electromagnetic Field of a Steady Charge Flow

A *stationary charge flow* is a continuous distribution of moving charge with time-independent charge density ρ_E and current density $\mathbf{J_E}$ in a given IRF.

At a point P, the current density $\mathbf{J_E}$ is defined such that its flux through an oriented surface **dS** gives the rate of charge transport through that surface. Thus,

$$i_E = \iint \mathbf{J_E}\cdot\mathbf{dS}\,.$$

During *dt*, the charge flow through an oriented surface element **dS** sweeps out the volume

$$dV = (\mathbf{v}\cdot\mathbf{dS})dt.$$

Hence

$$dq = \rho_E\ (\mathbf{v}\cdot\mathbf{dS})dt$$

and comparison with $di_E = \mathbf{J_E} \cdot \mathbf{dS}$ gives

$$\mathbf{J_E} = \rho_E \mathbf{v}.$$

A stationary charge flow may be treated as a continuous superposition of moving charge elements. Since ρ_E and $\mathbf{J_E}$ are time-independent, *the resulting electromagnetic field is stationary.*

1) Consequently, the electrostatic field associated with a stationary charge flow satisfies the same relations as an electrostatic electric field, namely (§4.5. and 4.6.)

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho_E}{\varepsilon_0} \qquad \text{and} \qquad \boldsymbol{\nabla} \times \mathbf{E} = 0$$

2) Since the b-index has no independent source, the corresponding b-information has no sources or sinks. Accordingly, the net b-information flux through any closed surface vanishes:

$$\oiint \mathbf{B} \cdot \mathbf{dS} = 0$$

By Ostrogradsky's divergence theorem [9], this is equivalent to

$$\nabla \cdot \mathbf{B} = 0.$$

As a result, the magnetostatic field can be expressed as the curl of a vector potential **A** [9]:

$$\mathbf{B} = \nabla \times \mathbf{A}.$$

Finally consider a surface ΔS with contour L and let $\mathbf{J_E}$ be the electric current density at an arbitrary point P of ΔS. The electric current encircled by L is

$$i_{in} = \iint \mathbf{J_E} \cdot \mathbf{dS}\,.$$

For stationary currents, Ampère's law states that the circulation of the magnetic field **B** around a closed path depends only on the total enclosed current i_{in}:

$$\oint \mathbf{B} \cdot \mathbf{dl} = \mu_0 \iint \mathbf{J_E} \cdot \mathbf{dS}. \tag{1}$$

According to Stoke's theorem [9]

$$\oint \mathbf{B} \cdot \mathbf{dl} = \iint (\boldsymbol{\nabla} \times \mathbf{B}).\mathbf{dS}. \tag{2}$$

Combining (1) and (2) gives:

$$\iint (\boldsymbol{\nabla} \times \mathbf{B}).\mathbf{dS} = \mu_0 \iint \mathbf{J_E} \cdot \mathbf{dS}$$

Since the surface is arbitrary, it follows locally that

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{J_E}.$$

The magnetostatic field associated with a stationary charge flow is therefore characterized by

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \qquad \text{and} \qquad \boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{J_E}$$

12. The Electromagnetic Field of a Uniformly Accelerated Point Charge

12.1. General Kinematic Extension of Rule C.1 of the Postulate of the Emission of Informatons

Chapter 6 showed that the radial orientation specified by Rule C.1 of the Postulate of the Emission of Informatons is the special case corresponding to a source at rest in its proper IRF. For a source in uniform rectilinear motion, the Lorentz transformation of the e-information flux

shows that the e-index remains aligned with the local informaton trajectory in the co-moving frame of the source. The orientation of the e-index is therefore a kinematic property of the emission process rather than a universal radial property in all inertial reference frames.

The same kinematic principle extends naturally to accelerated motion. Although an accelerating source possesses no single inertial rest frame, its motion may be described as a continuous succession of momentarily co-moving inertial reference frames. In each of these frames, Rule C.1 remains locally valid. Consequently, *the orientation of the e-index at each emission event is determined by the local direction of the informaton trajectory in the corresponding momentarily co-moving frame.*

The following sections derive the consequences of this generalized kinematic orientation rule for the electromagnetic field generated by an accelerating charge.

12.2. The Emission of Informatons by a Uniformly Accelerated Point Charge

Consider a (positive) point charge q (fig. 5) undergoing rectilinear motion with constant acceleration relative to the IRF **O**. The analysis is restricted to a time interval during which the motion remains non-relativistic, so that $v \ll c$.

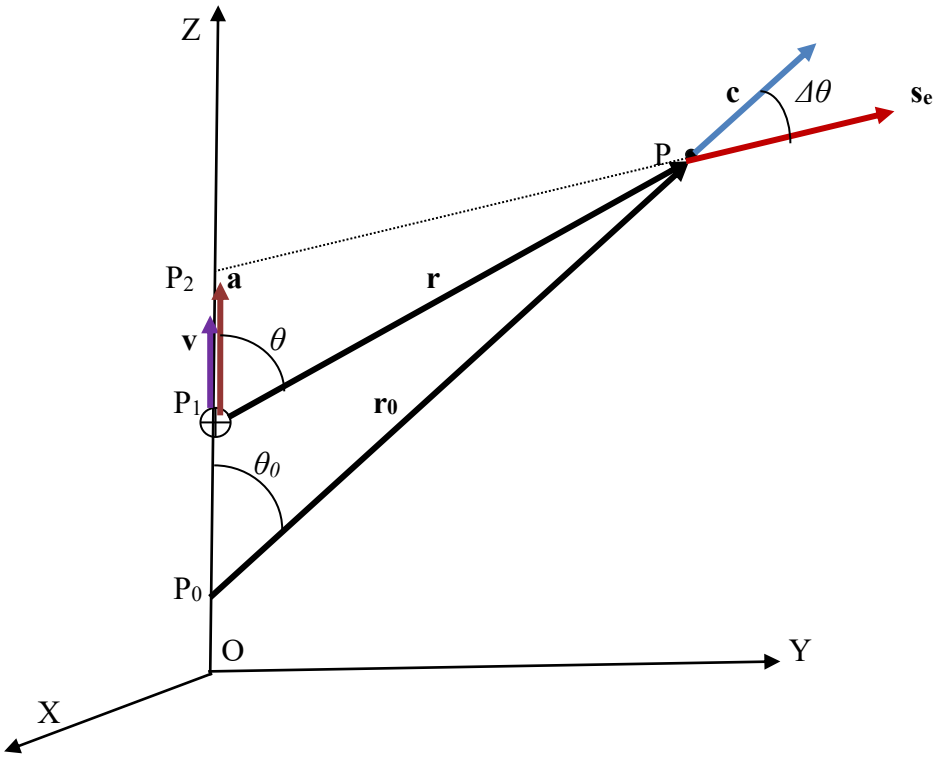


Figure 5. The emission of informatons by an accelerated point charge

The acceleration $\mathbf{a} = a\,\mathbf{u_z}$ is directed along the Z-axis. The charge is initially at rest at the origin O at time $t = 0$. After a finite time interval t, it reaches point P_1. At that instant, its velocity is $\mathbf{v}(t) = at\,\mathbf{u_z}$, and its position along the Z-axis is $z(t) = \frac{1}{2}at^2$.

Let P be a fixed observation point in space. Its instantaneous position relative to the charge at time t is represented by the vector **r**. The informatons that pass through P during the infinitesimal time interval $(t, t + \mathrm{d}t)$ were emitted at the corresponding *retarded time* $t_0 = t - \Delta t$, when the charge was located at P_0. The velocity of the charge at emission is therefore

$$\mathbf{v_0} = \mathbf{v}(t - \Delta t) = \mathbf{v} - \mathbf{a}\,\frac{r_0}{c}.$$

Here, $\mathbf{r_0} = \mathbf{P_0P}$ denotes the position vector of the observation point P relative to the retarded source position, **v** is the instantaneous velocity of the charge at time t and $\Delta t = r_0/c$ is the propagation time of the informatons from P_0 to P.

During Δt, the charge moves from P_0 to P_1, combining a uniform motion at velocity $\mathbf{v_0}$ with a constant acceleration **a**. The purpose of the following construction is to determine the effective geometric displacement that governs the orientation of the e-index at the observation point. We first consider the contribution arising from the retarded velocity and subsequently the additional contribution generated by the acceleration.

1) When the charge moves with *constant velocity* $\mathbf{v_0}$ along the Z-axis (fig 5,a), it moves from P_0 to P_1' during the interval Δt:

$$P_0P_1' = v_0\Delta t = v_0\,\frac{r_0}{c}.$$

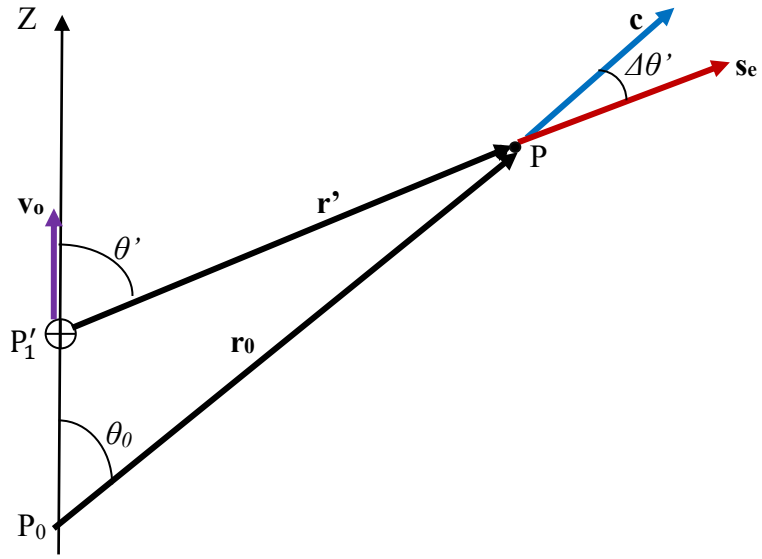


Figure 5,a. q is moving with constant speed v_0

The informatons crossing P at time t propagate at speed c along $\mathbf{r_0}$. According to the general kinematic extension of Rule C.1, their e-index is aligned with the local direction of the informaton trajectory as defined in the corresponding momentarily co-moving frame of the source. For the special case of uniform motion, the general kinematic rule reduces to the result

established in Chapter 6: the e-index is aligned with the vector $\mathbf{r}'$ joining the instantaneous position P_1' of the source to the observation point P.

2) When the charge starts from rest at P_0 and *accelerates uniformly* with $\mathbf{a} = a\mathbf{u_z}$, then during the same interval Δt it moves to P_1'' (fig 5,b):

$$P_0P_1'' = \frac{1}{2}a(\Delta t)^2 = \frac{1}{2}a\,\frac{r_0^2}{c^2}.$$

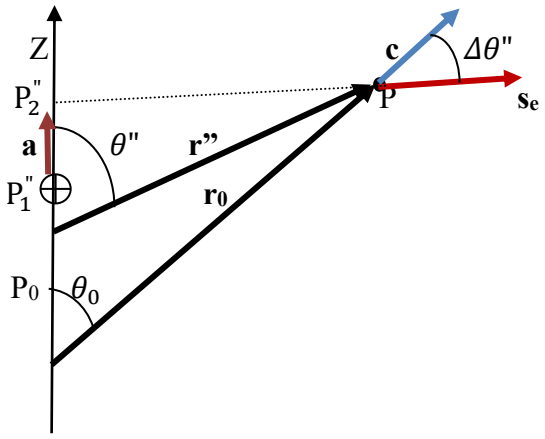


Figure 5,b. q is moving with constant acceleration **a**

The informatons reaching P at time t were emitted from the *retarded source position* P_0 and propagate at the invariant speed c along the radial direction defined by the vector $\mathbf{r_0}$.

According to the general kinematic extension of Rule C.1, the e-index at P is aligned with the tangent to the apparent informaton trajectory in the co-moving accelerating frame O′X′Y′Z′ (fig 5,c).

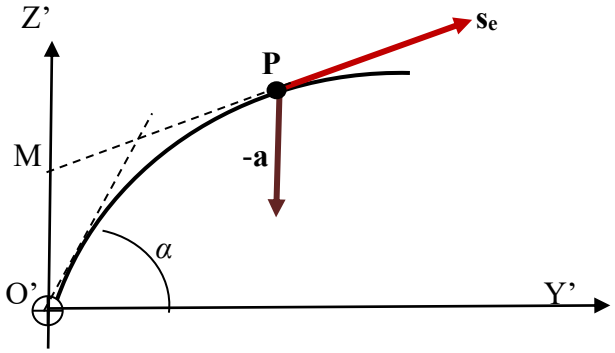


Figure 5 c. Apparent trajectory of an informaton in the accelerating frame O′X′Y′Z′.

In that frame, an informaton appears to undergo the kinematical acceleration $-\mathbf{a}$, and its apparent trajectory is the parabola [3] described by

$$z' = tg\alpha\; y' - \frac{1}{2}\frac{a}{c^2\,cos^2\,\alpha}.y'^2,$$

where $\alpha = \frac{\pi}{2} - \theta_0$. The curved form does not arise from any physical acceleration of the informaton. It arises solely from describing a rectilinear trajectory as observed from a uniformly accelerating reference frame.

At time $t = t_0 + \Delta t$, when the informaton reaches P, the tangent to its apparent trajectory intersects the Z′-axis at the point M. The position of M in the co-moving frame is given by

$$O'M = \frac{1}{2}a(\Delta t)^2 = \frac{1}{2}a\frac{r_0^2}{c^2},$$

where O' denotes the instantaneous position of the source in this frame.

The point M thus provides the geometric reference that determines the orientation of the e-index in the IRF construction. According to the general kinematic extension of Rule C.1, the e-index is aligned with the tangent, and therefore points along the vector from M to P. In the IRF construction (fig. 5b), the point M corresponds to P_2''whereas the instantaneous source position, represented by the origin O′, corresponds to P_1''. The displacement between these two points therefore is given by

$$P_1''P_2'' = O'M = \frac{1}{2}a\frac{r_0^2}{c^2}.$$

It follows immediately from fig 5,b that:

$$P_0P_2'' = P_0\ddot{P}_1 + P_1''P_2'' = \frac{1}{2}a\,\frac{r_0^2}{c^2} + \frac{1}{2}a\,\frac{r_0^2}{c^2} = a\,\frac{r_0^2}{c^2} = a(\Delta t)^2$$

Thus, the informatons reaching P at time propagate at the invariant speed along $\mathbf{P_0P}$, while their e-indices are directed along $\mathbf{P_2\ddot{P}}$.

3) In the IRF **O** (fig. 5), the combined effects of the instantaneous velocity and acceleration of the source charge determine an *effective geometric reference point* that specifies the orientation of the e-index. At time t, this reference point is located at P_2, whose position relative to P_0 is given by

$$P_0P_2 = \frac{v_0}{c}r_0 + \frac{a}{c^2}r_0^2.$$

This expression shows that the orientation of the e-index is governed by two distinct geometric contributions: one proportional to the retarded velocity of the source and the other proportional to its retarded acceleration.

As demonstrated here for the case of uniform accelerated motion, the e-index at a spacetime point is determined by the retarded kinematic state of the source and propagates outward at speed c. Spatial variation of its orientation at fixed time reflects the emission from different retarded source positions rather than intrinsic rotation during propagation. Temporal variation at a fixed observation point results from the successive arrival of configurations associated with different stages of the source's motion.

The relative importance of the velocity and acceleration contributions to the orientation of the e-index depends on the observation distance. If $r_0 \ll v_0c/a$, the acceleration contribution is negligible and the effective geometric reference point is determined primarily by the retarded velocity term. Conversely, if $r_0 \gg v_0c/a$, the acceleration contribution becomes dominant and determines the effective geometric reference point.

12.3. The Generation of the Electromagnetic Field by an Accelerated Point Charge

The informatons emitted during the infinitesimal interval $d\tau$ about the retarded time t_0 , when the source is located at P_0 , constitute a thin spherical shell centered at P_0 . The shell propagates outward at the invariant speed *c* and, at observation time *t*, has radius

$$r_0 = c\,\Delta t \qquad \text{with} \qquad \Delta t = t - t_0.$$

It carries the e-information emitted at the corresponding retarded time, with the e-index orientation determined by the effective geometric reference point P_2 derived in §12.2:

$$z_{p_2} - z_{p_0} = P_0P_2 = \frac{v_0}{c}\, r_0 + \frac{a}{c^2}\, r_0^2 = v_0\,\Delta t + a\,(\Delta t)^2. \qquad (1)$$

Space surrounding the charge may therefore be regarded as a nested succession of expanding shells associated with successive retarded source states. The electromagnetic field at an observation point is determined by the informational content and index geometry of the shell passing through that point.

12.4. The e-Indices of an Informaton emitted by an Accelerated Point Charge

Since the propagation direction is defined by the vector $\mathbf{P_0P}$ whereas the e-index orientation is represented by the vector $\mathbf{P_2P}$, the angle between both directions is determined by the triangle P_0P_2P (fig 5) . Applying the sine rule gives

$$\frac{\sin(\Delta\theta)}{P_0P_2} = \frac{\sin(\theta_0 + \Delta\theta)}{r_0}. \qquad (2)$$

Combining (1) and (2) yields

$$\sin\,(\Delta\theta) = \frac{v_0}{c}\sin(\theta_0 + \Delta\theta) + \frac{a}{c^2} r_0 \sin(\theta_0 + \Delta\theta).$$

In the non-relativistic regime, the source displacement during the propagation interval P_0P_1 is small compared with the observation distance P_0P Consequently, to first order,

$$r_0 \simeq r, \qquad \theta_0 + \Delta\theta \simeq \theta.$$

Therefore

$$\sin(\Delta\theta) \cong \frac{v_0}{c}\sin\,\theta + \frac{ar}{c^2}\sin\,\theta.$$

Let $(\mathbf{u_c}, \mathbf{u}_{\perp\mathbf{c}}, \mathbf{u}_\varphi)$ be a right-handed orthonormal triad with $\mathbf{u}_c = \mathbf{c}/c$. The unit vector $\mathbf{u}_{\perp c}$ is perpendicular to $\mathbf{u_c}$, lies in the plane spanned by **c** and r, and is oriented such that the transverse component of $\mathbf{s_e}$ – for a positive source charge – is positive. The third unit vector is $\mathbf{u}_\varphi = \mathbf{u}_c \times \mathbf{u}_{\perp c}$.

1) For a positive charge, $\Delta\theta$ is small and therefore $\cos\,(\Delta\theta) \simeq +1$. For a negative charge, $\Delta\theta$ is close to π, so that $\cos\,(\Delta\theta) \simeq -1$. In both cases, within the non-relativistic regime considered here,

$$\cos\,(\Delta\theta) \simeq \frac{q}{Q},$$

Hence, the longitudinal component of $\mathbf{s_e}$ is

$$s_e \cos\,(\Delta\theta) \simeq s_e\,\frac{q}{Q}.$$

2) With the above orientation convention for $\mathbf{u}_{\perp\mathbf{c}}$, the transverse component is

$$s_e \sin(\Delta\theta) \simeq \frac{q}{Q} s_e \left(\frac{v_0}{c} \sin\theta + \frac{ar}{c^2} \sin\theta\right).$$

3) The electric index vector can be expressed in terms of its components parallel and perpendicular to the propagation direction:

$$\mathbf{s_e} = s_e \cos(\Delta\theta)\, \mathbf{u}_c + s_e \sin(\Delta\theta)\, \mathbf{u}_{\perp c}$$

Substituting the above approximations for the longitudinal and transverse components yields

$$\mathbf{s_e} = \frac{q}{Q} s_e\, \mathbf{u}_c + \frac{q}{Q} s_e \left(\frac{v_0}{c} sin\,\theta + \frac{ar}{c^2} sin\,\theta\right) \mathbf{u}_{\perp c}\,. \quad (3)$$

From the definition of the b-index (§ 7.1) it follows that

$$\mathbf{s_b} = \frac{q}{Q} \frac{s_e}{c} \left(v_0 + \frac{ar}{c}\right) sin\,\theta\, \mathbf{u}_\varphi. \quad (4)$$

Equations (3) and (4) show that the informatons carry both longitudinal and transverse electromagnetic information. The term proportional to v_0/c represents the contribution associated with uniform motion, whereas the acceleration-dependent term ar/c^2 generates the radiative contribution, whose macroscopic field scales as $1/r$, in agreement with the non-relativistic radiation term of the Liénard–Wiechert field.

12.5. The Electromagnetic Field of an Accelerated Point Charge

Using the e- and b-indices derived in §12.4, the electromagnetic field at P follows directly from the corresponding informaton flux and volume densities.

1) Let N denote the flux density of informatons at P. By definition

$$\mathbf{E} = N\, \mathbf{s_e}.$$

With (§4.1)

$$N = \frac{K m_0}{4\pi r^2}$$

and Eq. (3)

$$\mathbf{s_e} = \frac{q}{Q} s_e\, \mathbf{u}_c + \frac{q}{Q} s_e \left(\frac{v_0}{c} sin\,\theta + \frac{ar}{c^2} sin\,\theta\right) \mathbf{u}_{\perp c}$$

where (§3.1)

$$s_e = \frac{Q}{m_0} \frac{1}{K\varepsilon_0},$$

one obtains

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^2}\, \mathbf{u}_c + \left[\frac{q}{4\pi\varepsilon_0 c r^2}\, v_0 \sin\theta + \frac{\mu_0 q}{4\pi r}\, a \sin\theta\right] \mathbf{u}_{\perp c}$$

The first term represents the quasi-static Coulomb component ($\propto 1/r^2$), while the second contains both a velocity-dependent correction and a radiative term proportional to the acceleration.

2) Let n denote the local informaton volume density. By definition

$$\mathbf{B} = n\, \mathbf{s_b}.$$

With (§7.2)

$$n = \frac{Km_0}{4\pi cr^2}$$

and Eq. (4)

$$\mathbf{s_b} = \frac{q}{Q}\frac{s_e}{c}\left(v_0 + \frac{ar}{c}\right) sin\,\theta\ \mathbf{u}_\varphi$$

where (§3.1)

$$s_e = \frac{Q}{m_0}\frac{1}{K\varepsilon_0},$$

one obtains

$$\mathbf{B} = \left[\frac{\mu_0 q}{4\pi r^2}\ v_0 \sin\theta + \frac{\mu_0 q}{4\pi cr}\ a \sin\theta\right]\mathbf{u}_\varphi.$$

The above results were derived under the assumption of uniform acceleration and non-relativistic motion ($v \ll c$). These relations remain valid when the acceleration varies slowly in time. Over the propagation interval $\Delta t = r/c$, the source parameters may therefore be treated as approximately constant. Accordingly, the velocity and acceleration entering the field expressions must be evaluated at the retarded time.

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^2}\ \mathbf{u}_c + \left[\frac{q}{4\pi\varepsilon_0 cr^2}v\left(t - \frac{r}{c}\right) sin\,\theta + \frac{\mu_0 . q}{4\pi r}a\left(t - \frac{r}{c}\right) sin\,\theta\right]\mathbf{u}_{\perp c}$$

$$\mathbf{B} = \left[\frac{\mu_0 q}{4\pi r^2}v\left(t - \frac{r}{c}\right) sin\,\theta + \frac{\mu_0 q}{4\pi cr}a\left(t - \frac{r}{c}\right) . sin\,\theta\right]\mathbf{u}_\varphi$$

These expressions reproduce the standard non-relativistic decomposition of the Lienard–Wiechert fields into near-field and radiative contributions.

At distances r large compared with the characteristic scale of the charge's motion, the $1/r^2$ terms become negligible. The remaining *1/r* components describe an *electromagnetic wave* propagating with velocity **c** along $\mathbf{u_c}$:

$$\mathbf{E} = \frac{\mu_0 q}{4\pi r}a\left(t - \frac{r}{c}\right) sin\,\theta\ \ \mathbf{u}_{\perp c}$$

$$\mathbf{B} = \frac{\mu_0 q}{4\pi cr}a\left(t - \frac{r}{c}\right) sin\,\theta\ \mathbf{u}_\varphi$$

The fields are mutually perpendicular and transverse to the propagation direction. Thus, in the informational formulation, radiation corresponds to the outward propagation of the acceleration-dependent e- and b-information carried by the informatons.

12.6. The Electromagnetic Field of a Harmonically Oscillating Point Charge

Consider a point charge q executing simple harmonic motion along the Z-axis about the origin of the IRF **O**. Its velocity is given by

$$v(t) = V\cos\omega\, t,$$

where $\omega = 2\pi\nu$. The motion is assumed to remain non-relativistic throughout, so that the velocity amplitude satisfies $V \ll c$. The corresponding position and acceleration are given by

$$z(t) = \frac{V}{\omega}\cos\ \left(\omega t - \frac{\pi}{2}\right), \qquad a(t) = \omega V\cos\ \left(\omega t + \frac{\pi}{2}\right).$$

At time t, let the instantaneous position of the charge be denoted by P_1. Let P be a fixed observation point with position vector $\mathbf{r}$. If the oscillation amplitude V/ω is small compared with r, the distance between the instantaneous source position P_1 and the observation point P may be approximated by

$$\mathbf{r_1} = \mathbf{P_1P} \cong \mathbf{OP} = \mathbf{r}.$$

Let θ denote the angle between the propagation direction $\mathbf{u_r}$ and the positive Z-axis.

The electromagnetic field at the observation point P depends on the retarded velocity and acceleration of the source, $v(t - r/c)$ and $a(t - r/c)$. Using the general result derived in §12.4 for non-relativistic motion, we derive that the magnetic field and the time-dependent part of the electric field at position P are given by

$$B_\varphi = \frac{E_{\perp c}}{c} = \frac{\mu_0 q}{4\pi r^2} v\left(t - \frac{r}{c}\right) sin\,\theta + \frac{\mu_0 q}{4\pi c r} a\left(t - \frac{r}{c}\right) sin\,\theta$$

$$= \frac{\mu_0 q}{4\pi r} V\, sin\,\theta \left[\frac{1}{r}\cos(\omega t - kr) - \frac{\omega}{c} sin\,(\omega t - kr)\right]$$

with $k = \omega/c$.

When $r \gg c/\omega = \lambda/2\pi$, with λ the wavelength, the velocity-dependent field term $\propto r^{-2}$ is negligible compared to the acceleration field term $\propto r^{-1}$. Hence, in the radiation zone:

$$B_\varphi^{(\text{far})} = -\frac{\mu_0\, q\, \omega\, sin\,\theta}{4\pi c r}\, V sin(\omega t - kr) \qquad (1)$$

or, using $a(t - r/c) = -\omega V \sin\,(\omega t - kr)$,

$$B_\varphi^{(\text{far})} = \frac{\mu_0 q\, sin\,\theta}{4\pi c r} a\left(t - \frac{r}{c}\right)$$

Since, in the radiation zone $E_{\perp c} = c\, B_\varphi$,

$$E_{\perp c}^{(\text{far})} = -\frac{\mu_0\, q\, \omega\, sin\,\theta}{4\pi r}\, V sin(\omega t - kr) = \frac{\mu_0 q\, sin\,\theta}{4\pi r} a\left(t - \frac{r}{c}\right) \qquad (2)$$

Equations (1) and (2) describe the transverse electromagnetic waves radiated by a harmonically oscillating point charge. In the radiation zone, the field amplitudes decrease in proportion to $1/r$, while their angular dependence is proportional to $\sin\theta$. The radiation-field amplitude is therefore determined solely by the component of the retarded acceleration perpendicular to the propagation direction,

$$a_\perp = a(t - r/c)\sin\,\theta.$$

This is the characteristic angular dependence of electric-dipole radiation and coincides with the classical result.

12.7. Far-zone Energy and Radiated Power

At an arbitrary observation point P located in the radiation zone, the transverse electric and magnetic fields derived in §12.6 are

$$E_{\perp c} = \frac{\mu_0 q \sin\theta}{4\pi r} a(t_r), \qquad B_\varphi = \frac{\mu_0 q \sin\theta}{4\pi c r} a(t_r),$$

with $t_r = t - r/c$.

The electromagnetic energy-flux density is represented by the Poynting vector [3]:

$$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B}).$$

Hence, at the observation point P, $\mathbf{S}$ is purely radial and given by

$$\mathbf{S} = \frac{E_{\perp c}\, B_\varphi}{\mu_0}\mathbf{u_c} = \frac{\mu_0 q^2 \omega^2 V^2 \sin^2\theta}{16\pi^2 c r^2} sin^2(\omega t - kr)\,\mathbf{u_c}$$

The differential quantity $d\mathcal{P}$ represents the power crossing a surface element $d\Sigma$ perpendicular to the direction of propagation

$$d\mathcal{P} = \frac{\mu_0 q^2 \omega^2 V^2 \sin^2\theta}{16\pi^2 c} sin^2(\omega t - kr)\, d\Omega \qquad \text{with} \qquad d\Omega = \frac{d\Sigma}{r^2}.$$

On a sphere of fixed radius r, the radiated power observed at time t depends on the retarded source time $t_r = t - r/c$. Equivalently, expressed as a function of the emission time, the instantaneous angular distribution is

$$\frac{d\mathcal{P}}{d\Omega} = \frac{\mu_0 q^2 \omega^2 V^2 \sin^2\theta}{16\pi^2 c} sin^2\omega t$$

Integration over the full solid angle yields the total instantaneous radiated power,

$$\mathcal{P}(t) = \frac{\mu_0 q^2 \omega^2 V^2}{6\pi c} sin^2\omega t = \frac{\mu_0 q^2 a^2(t)}{6\pi c} = \frac{q^2 a^2(t)}{6\pi\varepsilon_0 c^3}$$

which is the *non-relativistic Larmor formula* and agrees with classical electrodynamics.

For harmonic motion with velocity $v(t) = V\cos\,\omega t$, the acceleration is $a(t) = -\omega V \sin\,\omega t$. Denoting the acceleration amplitude by $A = \omega V$, the time-averaged radiated power becomes

$$\langle \mathcal{P} \rangle = \frac{\mu_0 q^2 A^2}{12\pi c} = \frac{q^2 A^2}{12\pi \varepsilon_0 c^3}.$$

This expression therefore gives the average power radiated by a point charge undergoing non-relativistic harmonic motion. With the center of oscillation chosen as the origin, the electric dipole moment is $p(t) = qz(t)$. Since $\ddot{p}(t) = qa(t)$, the radiated power is proportional to the square of the second time derivative of the dipole moment, in agreement with the classical theory of electric-dipole radiation. The present informational formulation thus reproduces the classical description of both the radiation field and the associated transport of electromagnetic energy.

13. Maxwell's Equations

Consider a point charge q in general motion, generating an electromagnetic field in vacuum through the continuous isotropic emission of informatons in its instantaneous rest frame. At a spacetime point P, let *N* denote the local informaton flux density and *n* the corresponding volume density. Since the informatons propagate at speed *c*, these quantities are related by

$$n = \frac{N}{c}. \tag{1}$$

Each informaton carries two source-dependent attributes: an electric index $\mathbf{s_e}$, representing the orientation of the electric information transported by the informaton, and a magnetic index $\mathbf{s}_\mathrm{b}$, arising from the motion of the source. The magnetic index is defined by

$$\mathbf{s_b} = \frac{\mathbf{c} \times \mathbf{s}_\mathrm{e}}{c} = \mathbf{u_c} \times \mathbf{s_e}, \tag{2}$$

where $\mathbf{c}$ denotes the propagation velocity of the informaton, and $\mathbf{u_c} = \mathbf{c}/c$ the unit vector in the propagation direction.

The contribution of the source to the local electric field at P is defined as the e-information flux density,

$$\mathbf{E} = N\mathbf{s_e}, \tag{3}$$

while the magnetic field is defined as the local density of b-information,

$$\mathbf{B} = n\mathbf{s_b}. \tag{4}$$

1) In the following analysis, the motion of the source is assumed to be non-relativistic ($v \ll c$). Consider the thin shell segment formed by the informatons emitted into the infinitesimal solid angle dΩ during the emission interval (t_0, $t_0 + d\tau$). The shell propagates away from the source with velocity $\mathbf{c} = c\,\mathbf{u_c}$. Its thickness, $d\ell = c\,d\tau$, remains constant during propagation, whereas its cross-sectional area normal to the local propagation direction is $s^2\,d\Omega$, where *s* denotes the distance measured along the axis of the conical propagation tube. Within the local observation domain introduced below, *s* also serves as the local longitudinal coordinate along $\mathbf{u_c}$.

In the following sections, the analysis is restricted to *a local observation domain* surrounding an observation point P. Over the infinitesimal propagation interval considered, the change in the cross-sectional area of the conical propagation tube is neglected, so that the tube may be treated

as locally cylindrical. Within this local approximation, both the informaton volume density n and the informaton flux density N are regarded as uniform across each cross section.

Variations in the propagation directions of the informatons constituting the shell segment are likewise neglected within the local observation domain. Their propagation directions may therefore be represented by a common local direction $\mathbf{u_c}$, so that the local propagation triad $(\mathbf{u_c}, \mathbf{u_{\perp c}}, \mathbf{u_\varphi})$, may also be regarded as constant within this domain. Variations in s consequently represent local displacements along $\mathbf{u_c}$.

Consider the succession of thin shell segments that pass through the observation point P. Each shell segment corresponds to a successive retarded state of the source and is formed by the informatons emitted during an equal emission interval $d\tau$ into the corresponding infinitesimal solid-angle element dΩ. Once emitted, a given shell segment propagates independently of the subsequent motion of the source and remains associated with the source position and state at its retarded emission time. During the infinitesimal time interval dt in which the segment propagates from P to the neighboring point Q over the distance $ds = cdt$, the source moves only a distance vdt. Since $v \ll c$, this displacement is small compared with ds, so that the source position relevant to the local propagation geometry at P may be regarded as effectively fixed over this interval.

Within the local cylindrical approximation, the change in the cross-sectional area of the shell segment during its propagation from P to Q is neglected. Since its thickness remains constant, the volume of the corresponding locally cylindrical shell element is likewise regarded as unchanged. Differences between the propagation directions of successive shell segments as they pass through P are also neglected within the same approximation, so that the segments may be treated as propagating through the local observation domain in the common direction $\mathbf{u_c}$.

Consider in particular the shell segment that passes through the observation point P, located at propagation coordinate s, at time t. Let $n(t, s)$ denote the corresponding local informaton volume density, which is determined by the retarded state of the source associated with that particular shell segment. After an infinitesimal time interval dt, this same shell segment has propagated to the neighboring point Q, located at coordinate $(s + ds)$, where $ds = c\,dt$.

At that later time, however, P is occupied by the next shell segment in the succession. The value $n(t + dt, s)$ at P is therefore associated with a slightly later retarded state of the source than the value $n(t, s)$. Conversely, the shell segment that was present at P at time t has meanwhile reached Q and determines the value $n(t + dt, s + ds)$ there.

Thus, the temporal difference between $n(t + dt, s)$ and $n(t, s)$ at the fixed observation point P reflects the difference between two successive retarded source configurations, whereas $n(t, s)$ and $n(t + dt, s + ds)$ refer to the same shell segment followed during its propagation from P to Q. The number of informatons carried by this segment is conserved. Since, within the local cylindrical approximation, both its thickness and its cross-sectional area are regarded as unchanged during the infinitesimal propagation from P to Q, its volume is likewise regarded as unchanged over this interval. Consequently, its informaton volume density satisfies

$$n(t, s) = n(t + dt, s + ds).$$

Hence,

$$n(t + dt, s + ds) - n(t, s) = 0.$$

To first order, this relation gives

$$\frac{\partial n}{\partial t} dt + \frac{\partial n}{\partial s} ds = 0$$

Using $ds = c\, dt$, one obtains

$$\frac{\partial n}{\partial t} + c \frac{\partial n}{\partial s} = 0 \tag{5}$$

This is *the continuity relation for the informaton volume density* within the local cylindrical approximation.

2) Consider the succession of thin shell segments propagating through the local observation domain introduced above. Each shell segment is associated with a definite state of the source at its corresponding retarded emission time and carries the informaton configuration determined by that state.

Let $F(t, s)$ denote a relevant quantity characterizing the informaton configuration present at propagation coordinate s at time t. Consider the shell segment that passes through the observation point P, located at coordinate s, at time t. The value $F(t, s)$ is determined by the retarded state of the source associated with that particular shell segment. After an infinitesimal time interval dt, this same shell segment has propagated to the neighboring point Q, located at coordinate $(s + ds)$, where $ds = c\, dt$.

At that later time, however, P is occupied by the next shell segment in the succession. The value $F(t + dt, s)$ at P is therefore associated with a slightly later retarded state of the source than the value $F(t, s)$. Conversely, the shell segment that was present at P at time t has meanwhile reached Q and determines the value $F(t + dt, s + ds)$ there.

Thus, the temporal difference between $F(t + dt, s)$ and $F(t, s)$ at the fixed observation point P reflects the difference between two successive retarded source configurations, whereas $F(t, s)$ and $F(t + dt, s + ds)$ refer to the same shell segment followed during its propagation from P to Q. For the transported quantities considered below, the value of F at P already incorporates any evolution accumulated during propagation from the corresponding retarded source state to P. For these transported quantities, any additional change in F while the same shell segment propagates from P to Q is neglected within the local approximation. Consequently,

$$F(t, s) = F(t + dt, s + ds).$$

Hence,

$$F(t + dt, s + ds) - F(t, s) = 0.$$

To first order,

$$\frac{\partial F}{\partial t} dt + \frac{\partial F}{\partial s} ds = 0$$

Using $ds = c\, dt$, one obtains

$$\frac{\partial F}{\partial t} + c \frac{\partial F}{\partial s} = 0 \tag{6}$$

This is *the propagation relation for those transported informaton quantities to which the local approximation applies*. It relates the temporal variation at a fixed observation point, caused by the arrival of successive retarded source configurations, to the corresponding spatial variation produced by their propagation through the local observation domain.

3) Within the local cylindrical approximation introduced in point 1, transverse spatial variations across each cross section are neglected, and the local triad $(\mathbf{u_c}, \mathbf{u_{\perp c}}, \mathbf{u_\varphi})$ may be regarded as constant within the local observation domain. Spatial variations of the quantities considered here therefore occur only along the propagation coordinate s.

For a scalar quantity $F(t, s)$, it follows that

$$\boldsymbol{\nabla} F = \mathbf{u_c} \frac{\partial F}{\partial s}.$$

Similarly, for a vector quantity $\mathbf{A}(t, s)$, whose components are expressed in the local propagation triad $(\mathbf{u_c}, \mathbf{u_{\perp c}}, \mathbf{u_\varphi})$ and vary only along s,

$$\boldsymbol{\nabla} \cdot \mathbf{A} = \mathbf{u_c} \cdot \frac{\partial \mathbf{A}}{\partial s}$$

and

$$\boldsymbol{\nabla} \times \mathbf{A} = \mathbf{u_c} \times \frac{\partial \mathbf{A}}{\partial s}.$$

These relations will be used below when evaluating the divergence and curl of the informational field quantities.

13.1. Maxwell-Gauss equation and its corollary

In the informational framework, Gauss's law is the local expression of *the conservation of e-information* established in §4 and §6.

At a regular spacetime point, where the electric charge density is ρ_E, it takes the form

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho_E}{\varepsilon_0}. \tag{7}$$

In free space, where $\rho_E = 0$,

$$\boldsymbol{\nabla} \cdot \mathbf{E} = 0.$$

We now derive a *local corollary of the source-free Maxwell–Gauss equation* expressed in terms of the local informaton variables N and $\mathbf{s}_\mathrm{e}$.

Using (3)

$$\boldsymbol{\nabla} \cdot \mathbf{E} = (\boldsymbol{\nabla} N) \cdot \mathbf{s_e} + N\,(\boldsymbol{\nabla} \cdot \mathbf{s_e}). \tag{8}$$

The two terms are examined separately.

1) Since $N = cn$ and c is constant, the continuity relation (5) also applies to N. Together with the local gradient relation established in point 3, this gives

$$\frac{\partial N}{\partial s} = -\frac{1}{c}\frac{\partial N}{\partial t},$$

and therefore

$$\boldsymbol{\nabla} N = -\frac{1}{c}\frac{\partial N}{\partial t}\mathbf{u_c}$$

Using

$$\mathbf{u_c} \cdot \mathbf{s_e} = s_e \cos(\Delta\theta),$$

one obtains

$$(\boldsymbol{\nabla} N) \cdot \mathbf{s_e} = -\frac{s_e}{c}\frac{\partial N}{\partial t}\cos(\Delta\theta). \tag{9}$$

2) Applying the local divergence relation established in point 3 to the e-index gives

$$\boldsymbol{\nabla} \cdot \mathbf{s_e} = \frac{\partial[s_e \cos(\Delta\theta)]}{\partial s}.$$

Since s_e is a universal constant, the propagation-dependent variation of the longitudinal component is entirely determined by the characteristic deviation . Applying the propagation relation (6) to this component gives

$$\frac{\partial[s_e \cos(\Delta\theta)]}{\partial s} = -\frac{1}{c}\frac{\partial[s_e \cos(\Delta\theta)]}{\partial t}$$

Therefore

$$N\,(\boldsymbol{\nabla} \cdot \mathbf{s_e}) = -\frac{N}{c} s_e \frac{\partial[\cos(\Delta\theta)]}{\partial t}. \tag{10}$$

3) Substituting Eqs. (9) and (10) into (8) yields

$$\boldsymbol{\nabla} \cdot \mathbf{E} = -\frac{s_e}{c}\frac{\partial}{\partial t}[N \cos(\Delta\theta)] \tag{11}$$

Since Gauss's law in free space requires

$$\boldsymbol{\nabla} \cdot \mathbf{E} = 0,$$

Eq. (11) implies

$$\frac{\partial}{\partial t}[N \cos(\Delta\theta)] = 0. \tag{12}$$

Thus, within the local far-field approximation considered here, the source-free Maxwell–Gauss equation is expressed as the local conservation of the longitudinal component of the e-information flux carried by the propagating informaton structure.

13.2. Maxwell-Gauss equation for magnetism

In the informational formulation, the b-index is not independently assigned but is defined kinematically by the propagation geometry of the informatons. The divergence of the resulting magnetic field is obtained using Eq.4

$$\boldsymbol{\nabla} \cdot \mathbf{B} = (\boldsymbol{\nabla} n) \cdot \mathbf{s_b} + n\,(\boldsymbol{\nabla} \cdot \mathbf{s_b}).$$

The two terms are examined separately.

1) Using the spatial relation established in point 3,

$$\boldsymbol{\nabla} n = \frac{\partial n}{\partial \mathrm{s}}\mathbf{u_c}.$$

Since, by definition,

$$\mathbf{s_b} = \mathbf{u_c} \times \mathbf{s_e}$$

one has $\mathbf{s_b} \perp \mathbf{u_c}$.

Therefore,

$$(\boldsymbol{\nabla} n) \cdot \mathbf{s_b} = \mathbf{0}$$

2) Applying the local divergence relation established in point 3 to the b-index gives

$$\boldsymbol{\nabla} \cdot \mathbf{s_b} = \mathbf{u_c} \cdot \frac{\partial \mathbf{s_b}}{\partial s}.$$

Because $\mathbf{u_c}$ is regarded as constant within the local observation domain,

$$\mathbf{u_c} \cdot \frac{\partial \mathbf{s_b}}{\partial s} = \frac{\partial}{\partial s}(\mathbf{u_c} \cdot \mathbf{s_b}) = \mathbf{0}.$$

Since $\mathbf{s_b}$ is perpendicular to $\mathbf{u_c}$,

$$\mathbf{u_c} \cdot \mathbf{s_b} = \mathbf{0}.$$

Therefore,

$$\nabla \cdot \mathbf{s_b} = \mathbf{0}.$$

3) Substituting the above results into the divergence expression for **B** yields

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0. \tag{13}$$

The absence of magnetic monopoles therefore emerges, within the informational formulation, as a consequence of the kinematic definition of the b-index.

13.3. Maxwell–Faraday equation

Taking the curl of Eq. (3) gives

$$\boldsymbol{\nabla} \times \mathbf{E} = \boldsymbol{\nabla} \times (N \mathbf{s_e}) = (\boldsymbol{\nabla} N) \times \mathbf{s_e} + N\,(\boldsymbol{\nabla} \times \mathbf{s_e}). \tag{14}$$

The two contributions are examined separately.

1) Since $N = cn$ and c is constant, the continuity relation (5) also applies to N. Together with the local gradient relation established in point 3, this gives

$$\boldsymbol{\nabla} N = \frac{\partial N}{\partial s}\,\mathbf{u_c} = -\frac{1}{c}\frac{\partial N}{\partial t}\,\mathbf{u_c}$$

Hence

$$\boldsymbol{\nabla} N \times \mathbf{s_e} = -\frac{1}{c}\frac{\partial N}{\partial t}\,(\mathbf{u_c} \times \mathbf{s_e}). \tag{15}$$

Since $\mathbf{s_b} = (\mathbf{u_c} \times \mathbf{s_e})$ and $n = N/c$, with **c** constant, it follows that

$$\boldsymbol{\nabla} N \times \mathbf{s_e} = -\frac{\partial n}{\partial t}\,\mathbf{s_b}. \tag{16}$$

2) The local curl relation established in poi 3 gives

$$\boldsymbol{\nabla} \times \mathbf{s_e} = \mathbf{u}_c \times \frac{\partial \mathbf{s_e}}{\partial s}.$$

Equation (6), applied to the e-index, gives

$$\frac{\partial \mathbf{s_e}}{\partial s} = -\frac{1}{c}\frac{\partial \mathbf{s_e}}{\partial t}$$

Hence, using $\mathbf{s}_b = \mathbf{u}_c \times \mathbf{s}_e$, where $\mathbf{u}_c$ is regarded as constant over the local spacetime domain, one obtains

$$\boldsymbol{\nabla} \times \mathbf{s_e} = -\frac{1}{c}\frac{\partial \mathbf{s_b}}{\partial t}.$$

Multiplying both sides by N, and using $n = N/c$, yields

$$N(\boldsymbol{\nabla} \times \mathbf{s_e}) = -\, n\, \frac{\partial \mathbf{s_b}}{\partial t}. \tag{17}$$

3) Substituting Eq. (16) and Eq. (17) into Eq. (14) gives

$$\boldsymbol{\nabla} \times \mathbf{E} = -\, \frac{\partial}{\partial t}(n\, \mathbf{s_b}).$$

Using Eq. 4 this yields

$$\boldsymbol{\nabla} \times \mathbf{E} = -\, \frac{\partial \mathbf{B}}{\partial t}. \tag{18}$$

Within the local approximation introduced above, one thus recovers the classical Maxwell–Faraday equation, relating the temporal variation of the b-information density to the spatial circulation of the e-information flux density.

13.4. Maxwell-Ampère equation

Taking the curl of Eq. (4)

$$\boldsymbol{\nabla} \times \mathbf{B} = [(\boldsymbol{\nabla} n) \times \mathbf{s_b}] + n(\boldsymbol{\nabla} \times \mathbf{s_b}), \tag{19}$$

The two contributions are examined separately.

1) Combining the continuity relation (5) with the local gradient relation established in point 3 gives,

$$\boldsymbol{\nabla} n = \frac{\partial n}{\partial s}\, \mathbf{u_c} = -\frac{1}{c}\frac{\partial n}{\partial t}\, \mathbf{u_c}.$$

Hence

$$(\boldsymbol{\nabla} n) \times \mathbf{s_b} = -\frac{1}{c}\frac{\partial n}{\partial t}(\mathbf{u_c} \times \mathbf{s_b}).$$

Using

$$\mathbf{s_b} = \mathbf{u_c} \times \mathbf{s_e}$$

and the decomposition

$$\mathbf{s_e} = s_e \cos(\Delta\theta)\mathbf{u_c} + s_e \sin(\Delta\theta)\mathbf{u_{\perp c}},$$

the vector triple-product identity gives

$$\mathbf{u_c} \times \mathbf{s_b} = -s_e \sin(\Delta\theta)\, \mathbf{u_{\perp c}}.$$

Since $n=N/c$, it follows that

$$(\boldsymbol{\nabla} n) \times \mathbf{s_b} = \frac{1}{c^2}\frac{\partial N}{\partial t} s_e \sin(\Delta\theta)\, \mathbf{u_{\perp c}}. \tag{20}$$

2) Applying the propagation relation (6) to the b-index gives

$$\frac{\partial \mathbf{s_b}}{\partial s} = -\frac{1}{c}\frac{\partial \mathbf{s_b}}{\partial t}.$$

Using the curl relation established in point 3,

$$\boldsymbol{\nabla} \times \mathbf{s_b} = \mathbf{u_c} \times \frac{\partial \mathbf{s_b}}{\partial s} = -\frac{1}{c}\, \mathbf{u_c} \times \frac{\partial \mathbf{s_b}}{\partial t}.$$

Since the local basis vectors are regarded as constant, and

$$\mathbf{u_c} \times \mathbf{s_b} = -s_e \sin(\Delta\theta)\mathbf{u_{\perp c}}$$

it follows

$$n(\boldsymbol{\nabla} \times \mathbf{s_b}) = \frac{N}{c^2}\frac{\partial}{\partial t}[s_e \sin(\Delta\theta)\mathbf{u}_{\perp \mathbf{c}}]. \tag{21}$$

3) Adding Eqs. (20) and (21), the time derivative of the density and the time derivative of the transported index combine naturally into the derivative of their product.

$$\boldsymbol{\nabla} \times \mathbf{B} = \frac{1}{c^2}\frac{\partial}{\partial t}[N s_e \sin(\Delta\theta)\mathbf{u}_{\perp \mathbf{c}}]. \tag{22}$$

Using the decomposition of **E**,

$$\mathbf{E} = N s_e \cos(\Delta\theta)\,\mathbf{u_c} + N s_e \sin(\Delta\theta)\mathbf{u}_{\perp \mathbf{c}}$$

together with the free-space invariance relation (12), according to which the longitudinal e-information flux density is conserved in free space, one obtains

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{\partial}{\partial t}[N s_e \sin(\Delta\theta)\mathbf{u}_{\perp \mathbf{c}}].$$

Therefore,

$$\boldsymbol{\nabla} \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}. \tag{23}$$

As derived in §11, a stationary charge flow gives rise to the magnetostatic relation

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}_E. \tag{24}$$

The above derivation shows that, in addition, temporal variations of the e-information flux density contribute through the term

$$\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}.$$

For a general configuration, the magnetic field is the linear superposition of the contributions associated with charge transport and with the time-dependent free-space informaton distribution. Since the curl operator is linear, the corresponding contributions to $\nabla \times \mathbf{B}$ add, yielding

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}_E + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}. \tag{25}$$

The second term corresponds to the displacement-current contribution of classical electrodynamics. Within the local approximation introduced above, one thus recovers the Maxwell–Ampère equation. In the informational formulation, the circulation of the b-information density contains two contributions: one associated with electric current and the other with the temporal variation of the e-information flux density.

13.5. Conclusion

Within the informational formulation, the four Maxwell equations considered above acquire a direct interpretation in terms of the transport and superposition of informatons and the geometric and kinematic relations between their indices. Gauss's law expresses the local conservation of e-information emitted by electric charges; in free space, its corollary is the local conservation of the longitudinal component of the e-information flux. The absence of magnetic monopoles emerges from the kinematic definition of the b-index. The Maxwell–Faraday equation relates the spatial circulation of the e-information flux density to the temporal variation of the b-information density, while the Maxwell–Ampère equation combines the contribution associated with electric current with the displacement current contribution arising from the temporal variation of the e-

information flux density. Taken together, these results reproduce the classical differential Maxwell equations within the local, non-relativistic description adopted here.

14. The Interaction between Moving Point charges – Lorentz Force Law

14.1. The Electromagnetic Force on a Moving Charged Particle in an Electromagnetic Field

Let **O** denote an inertial reference frame in flat Minkowski spacetime. All field quantities (**E**, **B)**, particle velocities **v**, and relativistic momenta **p** are defined with respect to this frame. Time derivatives d/dt are taken with respect to the coordinate time of **O**.

Consider a set of point charges moving with respect to **O.** These charges generate an EM field described by the vectors **E** and **B**, defined at every point of the associated space. Each particle is immersed in the informational distribution generated by the informatons emitted by all particles. Except at its instantaneous position, where the self-field is singular, each charge contributes to this distribution throughout space.

Let us focus on a particular charge q which, at time t, passes the point P with velocity **v.**

1) In the absence of other particles, the electric self-field **E**s of q would, according to §6, be axially symmetric about the direction of the velocity **v**. In the presence of other charges, this symmetry is disturbed by the e-information emitted by them. Since **E** at P represents the local density of the e-information flux, it reflects the deviation from this axial symmetry induced by the surrounding charges.

2) Similarly, in the absence of other particles, the magnetic self-field $\mathbf{B_S}$ of q would, according to §7.3, exhibit the same axial symmetry. The associated vector field ($\mathbf{v} \times \mathbf{B_S}$), would share this symmetry. In the presence of other moving charges the b-information emitted by them modifies this magnetic symmetry, and consequently, the symmetry of the *pseudo-electric structure*. The vector product (**v** × **B**) at P reflects the deviation from this axial symmetry induced by the surrounding moving charges.

Because the informational disturbances produced by external e-information and b-information are independent and the electromagnetic field depends linearly on the corresponding informaton densities, their effects combine additively. The combined informational disturbance can therefore be represented by an *effective electric field* $\bar{\mathbf{E}}$ defined as

$$\bar{\mathbf{E}} = \mathbf{E} + (\mathbf{v} \times \mathbf{B}).$$

The dynamical postulate introduced in §5.1 can now be generalized to moving charges: *a disturbance of the symmetry of a moving charged particle's informational self-field by the external electromagnetic informational environment gives rise to a restoring tendency whose magnitude is proportional to the magnitude of the disturbance and to the strength of the particle's self-field.*

As established in Chapter 4, the strength of a particle's informational self-field is proportional to the magnitude of its charge. The resulting restoring tendency is therefore proportional to the magnitude of the effective electric field $\bar{\mathbf{E}}$ and to $|q|$, with its direction determined by the sign of

q relative to $\bar{\mathbf{E}}$. The electromagnetic force is accordingly defined by the simplest linear constitutive relation consistent with these properties:

$$\mathbf{F_{EM}} = q\bar{\mathbf{E}} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}).$$

The force defined above - *the Lorentz force* - represents the observable mechanical manifestation of the restoring tendency generated by the disturbance of the particle's informational self-field. The corresponding relativistic equation of motion is derived in the following section.

14.2. Motion of a Charged Particle in an Electromagnetic Field

To derive the relativistic equation of motion, we introduce in the particle's instantaneous rest frame **O'** a *virtual gravitational field* $\bar{\mathbf{E}}'_{\mathrm{g}}$, defined such that it produces the same mechanical effect as the external electromagnetic field. Since the particle is instantaneously at rest in **O'**, the magnetic contribution to the electromagnetic force vanishes, so that the force is determined solely by the external electric field $\mathbf{E}'$. Therefore,

$$m_0\bar{\mathbf{E}}'_{\mathrm{g}} = q\mathbf{E}'.$$

Hence,

$$\bar{\mathbf{E}}'_{\mathrm{g}} = \frac{q}{m_0}\mathbf{E}'$$

According to the local equivalence between a gravitational field and an accelerated reference frame, the mechanical action represented by the virtual gravitational field is locally equivalent to the acceleration of a freely moving particle. Consequently, the proper acceleration of the charged particle is

$$\mathbf{a}' = \frac{q}{m_0}\mathbf{E}'. \tag{1}$$

To express this relation in terms of quantities measured in the IRF **O** , the acceleration and electric field in **O'** are resolved into components parallel and perpendicular to the instantaneous velocity observed in **O**:

$$\mathbf{a}' = a'_T\mathbf{u_T} + a'_N\mathbf{u_N} \qquad \text{and} \qquad \mathbf{E}' = E'_T\mathbf{u_T} + E'_N\mathbf{u_N}.$$

1) The standard relativistic acceleration transformations give [10]

$$a'_T = \gamma^3 a_T \qquad \text{and} \qquad a'_N = \gamma^2 a_N,$$

where a_T and a_N denote the tangential and normal components of the acceleration measured in **O**, and

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}.$$

The corresponding Lorentz transformation of the electromagnetic field gives [10]

$$E'_T = E_T \qquad \text{and} \qquad E'_N = \gamma[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]_N. \tag{2}$$

Since $(\mathbf{v} \times \mathbf{B})$ is perpendicular to **v**, its tangential component vanishes, and therefore

$$E'_T = E_T = [\mathbf{E} + (\mathbf{v} \times \mathbf{B})]_T.$$

Combining Eq. (1) with the acceleration and field transformations then gives

$$m_0\gamma^3 a_T = q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]_T \tag{3}$$

and

$$m_0\gamma^2 a_N = q\gamma[\mathbf{E} + (\mathbf{v}\times\mathbf{B})]_N$$

or

$$m_0\,\gamma a_N = q[\mathbf{E} + (\mathbf{v}\times\mathbf{B})]_N. \qquad (4)$$

2) For the relativistic momentum

$$\mathbf{p} = \gamma m_0\mathbf{v}$$

the tangential and normal components of its time derivative are [10]

$$\left(\frac{\mathrm{d}\mathbf{p}}{\mathrm{dt}}\right)_T = m_0\gamma^3 a_T \qquad \text{and} \qquad \left(\frac{\mathrm{d}\mathbf{p}}{\mathrm{dt}}\right)_N = m_0\gamma a_N.$$

3) Equations (3) and (4) combine to give

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{dt}} = q\,[\mathbf{E} + (\mathbf{v}\times\mathbf{B})] = \mathbf{F_{EM}}.$$

The relativistic equation of motion thus follows from the same dynamical postulate introduced for the electrostatic interaction in Chapter 5. Within the informational framework, the Lorentz force represents the observable mechanical manifestation of the restoring tendency generated by the disturbance of the symmetry of a moving charged particle's informational self-field by its electromagnetic environment.

14.3. The Electromagnetic Interaction between two Moving Point Charges

Consider (fig 6) two point charges q_1 and q_2 at rest in the IRF $\mathbf{O'}$, which moves with constant velocity $\mathbf{v} = v.\mathbf{u_z}$ relative to IRF **O.**

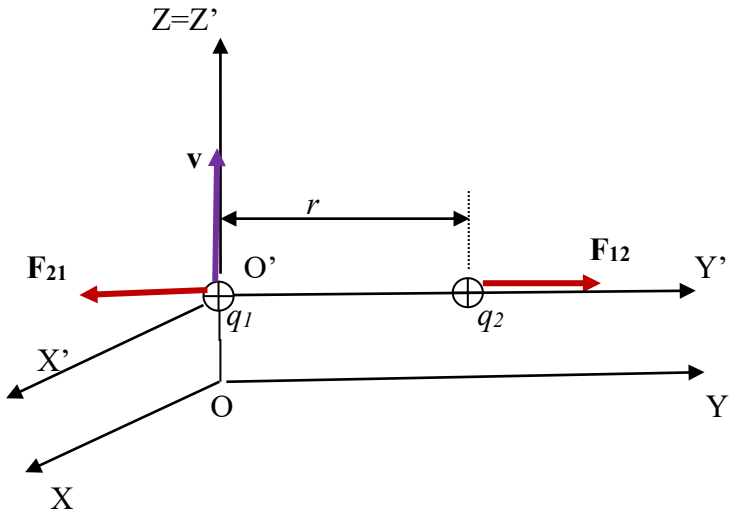


Figure 6. Interaction between two moving point charges

According to the field expressions derived in §8 for a charge moving uniformly perpendicular to the line joining source and observer, the magnitudes of the electric and magnetic field components generated by q_1 at the position of q_2 are

$$E_2 = \frac{q_1}{4\pi\varepsilon_0 r^2}.\frac{1}{\sqrt{1-\beta^2}} \qquad \text{and} \qquad B_2 = \frac{q_1}{4\pi\varepsilon_0 r^2}.\frac{1}{\sqrt{1-\beta^2}}.\frac{v}{c^2}$$

The vector $\mathbf{E_2}$ points along the positive Y-axis, whereas $\mathbf{B_2}$ points along the negative X-axis.

According to §14.1, the Lorentz force exerted by the EM field $(\mathbf{E_2}, \mathbf{B_2})$ on charge q_2 is:

$$\mathbf{F_{12}} = q_2[\mathbf{E_2} + (\mathbf{v} \times \mathbf{B_2})].$$

Given the orientation of the vectors in fig. 6, the force points along the Y-axis, and its magnitude is

$$F_{12} = q_2(E_2 - v.B_2).$$

Substituting the expressions for E_2 and B_2 yields

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\sqrt{1-\beta^2} = F'_{12}\sqrt{1-\beta^2},$$

where

$$F'_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

is the magnitude of the Coulomb force exerted by q_1 on q_2 in the IRF **O'** in which both particles are at rest.

By symmetry in the common rest frame **O′**, one likewise obtains

$$F_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\sqrt{1-\beta^2} = F'_{21}\sqrt{1-\beta^2}$$

Hence, the two moving point charges repel each other with equal and opposite forces of magnitude

$$F = F_{12} = F_{21} = F'\sqrt{1-\beta^2}.$$

Since the force is purely transverse to the relative motion of the frames, this result is consistent with the relativistic transformation law for transverse force components [10].

The magnetic contribution is smaller than the electric contribution by a factor of order β^2 and is therefore negligible for $v \ll c$. This differs from current-carrying conductors, where electric contributions largely cancel while the magnetic interaction remains.

14.4. The Electromagnetic Force on a Current Element in an Electromagnetic field.

In the neutral-conductor model considered here, the charge density of the mobile carriers is compensated by the oppositely charged stationary lattice, so that the electric contributions to the Lorentz force cancel macroscopically, while the magnetic contribution associated with the moving carriers remains. The Lorentz force on the charge dq carried by a current element therefore reduces to

$$\mathbf{dF} = dq\ (\mathbf{v} \times \mathbf{B})$$

Using $i\mathbf{dl} = \mathbf{v}dq$, this becomes

$$d\mathbf{F} = i(\mathbf{dl} \times \mathbf{B}).$$

Within the informational framework, this reflects the cancellation of the e-information contributions, whereas the b-information associated with the moving carriers is not compensated by the stationary lattice.

14.5. Interaction between two Parallel Current-Carrying Conductors

Figure 7 shows two long, parallel conductors anchored in an IRF, separated by a distance r and carrying steady currents i_1 and i_2.

According to the Biot–Savart relation derived in §10, the magnetic field produced by the left wire at the position of the right one has the magnitude

$$B_1 = \frac{\mu_0\, i_1}{2\pi r}$$

and, as indicated in the figure, its direction is perpendicular to the plane of the diagram.

The magnetic force exerted by this field on a segment of length L of the right wire is, according to the Lorentz force relation established in §14.4

$$F_{12} = i_2\, L\, B_1 = \frac{\mu_0\, i_1\, i_2}{2\pi r}\, L$$

and it acts to the left.

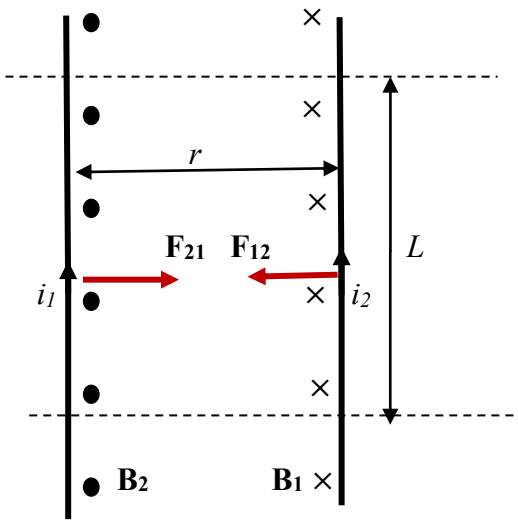


Figure 7. Interaction between two parallel current-carrying conductors

By symmetry the corresponding magnetic force on a segment of length L of the left wire is

$$F_{21} = i_1\, L\, B_2 = \frac{\mu_0\, i_1\, i_2}{2\pi r}\, L,$$

and directed to the right.

The forces are equal in magnitude and opposite in direction: parallel currents attract, whereas antiparallel currents repel. Their magnitude per unit length is

$$\frac{F}{L} = \frac{\mu_0\, i_1\, i_2}{2\pi r}.$$

This is *Ampère's force law*.

15. EPILOGUE

In this work, we have developed a discrete causal framework in which electromagnetic interactions are represented in terms of the emission and propagation of elementary informational carriers, termed informatons. From a small set of kinematic postulates, the framework reconstructs the Maxwell–Lorentz structure of classical electrodynamics in the continuum limit. In particular, it reproduces the classical electric and magnetic fields of point charges at rest and in

uniform motion, as well as the radiation field generated by accelerated charges. Within the local description developed in Chapter 13, the differential Maxwell equations in vacuum are recovered from the transport and superposition of informatons and from the geometric and kinematic relations between their indices.

The same framework also reproduces the classical interaction laws governing moving charges and current-carrying conductors, thereby completing the recovery of the Maxwell–Lorentz structure of classical electrodynamics.

Within this formulation, magnetic phenomena emerge as a kinematic consequence of the geometric non-collinearity between the propagation direction of an informaton and its electric index. The resulting description preserves relativistic causality, remains empirically equivalent to the conventional field formulation, and provides a discrete causal representation of electromagnetic interactions whose continuum limit coincides with the established theory.

Beyond reproducing the established Maxwell–Lorentz theory, the present formulation provides a unified causal interpretation of its principal laws in terms of the emission, propagation, and superposition of informatons. By reducing the electromagnetic interaction to a small number of kinematic and geometric principles, the framework may offer both a conceptually transparent interpretation of classical electrodynamics and a useful pedagogical perspective on the logical connections between its fundamental laws.

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