The principle of relativity in Newton's Principia and in today's classical physics

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Abstract
The relationship between Newtonian physics and the theory of relativity is investigated, with reference to Newton's masterwork. The tension within the Principia between Galilean relativity and a thoroughly relational concept of motion, such as the one espoused by Leibniz, is shown, and the way it surfaces also in contemporary physics textbooks is illustrated. The link between Newton's physics and its metaphysical underpinnings, which is central to the Leibniz-Clarke controversy, is clarified by investigating his cosmological assumptions.

***Keywords*:** Newtonian physics, Galilean relativity, Leibniz-Clarke correspondence, cosmological large structures, physics textbooks.

A recent (published) paper had near the beginning the passage 'The object of this paper is to prove (something very important).'It transpired with great difficulty, and not till near the end, that the 'object' was an unachieved one. *Littlewood's miscellany*.

1. Prologue

On perusing an acclaimed annotated English translation of Newton's masterwork [1], a bulky xvii+966 pages book provided with a very detailed analytical index, one may not fail to be surprised at discovering that the item 'principle of relativity' is nowhere listed – and, as to Albert Einstein (a byword for 'relativity' to most people), the only, indirect, reference occurs in its vast commentary in the section devoted to the issue of whether Newton's definition of "mass" is circular.

The main editor of this volume is I. Bernard Cohen, a famous historian of science, and co-editor of the critical Latin edition of *Philosophiae Naturalis Principia Mathematica* with one of the uppermost Newtonian scholars in the 20th century, Alexandre Koyré. Cohen also wrote a well-known outline of the scientific revolution [2], where he deals very briefly on "a form of relativity" by referring to Copernicus, and treating Galileo's contribution in the *Dialogue* very shortly and only insofar as the fall of bodies is concerned ("The usual reference frame was a moving ship" [p. 84]).

On the other hand, the importance of the principle of relativity in the Copernican controversy and Galileo's contribution to its establishment has featured as the main topic in numberless scholarly and popular accounts, from around 1920, in particular in Hermann Weyl's magnificent treatise first published in 1918, where the first section of chapter III is entitled "Das Galileische Relativitätsprinzip" [3].

What might be the source, then, of this curious silence in Cohen's book? Using this historiographical starting point, we shall elucidate a wider issue related both to teaching strategies (what can be termed as the 'physics textbooks' philosophy'), and to different views on scientific progress.

2. Galilean relativity and Newtonian force

In a contemporary approach to classical physics, the Galilean principle of relativity presents itself as a constraint on the form of the force function, based on the Galilei group [4].

The Galilean group is the subgroup of all affinities of R^4 of the form

$\mathbf{r}' = A(\mathbf{r} - t\mathbf{V}) + \mathbf{b}$, $t' = t + k$,

where $A \in SO(3)$, $\mathbf{b}, \mathbf{V} \in R^3$, $k \in R$. Every affine transformation of this form can be seen as the transition function between two admissible, inertial coordinate systems φ, φ'.

The second law of motion, or, as is commonly called today, the second principle of dynamics, states that a point particle of mass m obeys, in a given Galilean coordinate system φ, a second order ordinary differential equation in the normal form:

$$m\mathbf{a} = \mathbf{F}(\mathbf{r}, \mathbf{v}, t) \qquad (1)$$

If we assume that in every other φ' related to φ by a Galilean transformation a law of the very same type (i.e. with the same functional dependence) must hold:

$$m\mathbf{a}' = \mathbf{F}(\mathbf{r}', \mathbf{v}', t),$$

then a number of consequences follow about **F**. For instance, if there are only two particles, with masses m_1 and m_2, then the force law for the first one will be of the form

$$m_1\, \mathbf{a}_1 = \mathbf{f}\,(\mathbf{r}_1 - \mathbf{r}_2,\, \mathbf{v}_1 - \mathbf{v}_2) \qquad (2)$$

for a suitable function **f** satisfying, for all $A \in SO(3)$, and $\mathbf{r}, \mathbf{v} \in R^3$ the covariance condition:

$A\mathbf{f}\,(\mathbf{r},\mathbf{v}) = \mathbf{f}\,(A\mathbf{r}, A\mathbf{v})$.

If, moreover, the force function **F** is *positional*, i.e. independent of velocity, it is easy to prove that **f** must be directed as $\mathbf{r}_1 - \mathbf{r}_2$ and its module depend on the distance $|\mathbf{r}_1 - \mathbf{r}_2|$ only – that is, it is a *central* force, and the third principle of dynamics gives immediately, by a simple change of sign, also the force equation for the second particle.

Any interaction force of the form $g(|\mathbf{r}_1 - \mathbf{r}_2|)(\mathbf{r}_1 - \mathbf{r}_2)$ is, formally, a possible interaction force (of course other physical principles, such as some form of locality, will further reduce a priori the choice). The most famous example is of course Newton's attraction force, which clearly satisfies the principle of Galilean relativity. Thus Galilean relativity is at the core of the "physics of central forces", described by Henri Poincaré in his enlightening 1904 speech at Saint-Louis [5].

These statements can be generalized to any number of particles. If there are several interacting particles, the force **F** on a single one is the sum of all interaction forces with it (*superposition principle*), and in case **F** vanishes, the particle will move uniformly in a straight line – this is the law of inertia, also known as the first principle of dynamics.

So far we have presented an outline of the fundamentals of the subject in modern terms and formalism. Clearly some very important features of Newtonian physics are shaped by what we now call Galilean relativity. A natural question arises: is there any explicit statement on Galilean relativity (except for the name) in the *Principia*?

3. Two crucial corollaries in the Principia

The answer is yes. The principle of Galilean relativity is stated in the *Principia* as a corollary in Book I. Here is the statement with its proof (in the Motte-Cajori translation [6], which in this case has been only slightly modernized in [1]), referring to the by then standard ship example, which had been mentioned by Copernicus (1543), developed by Giordano Bruno (1584), and wonderfully expanded upon by Galileo in his *Dialogo* (1632):

Corollary V

The motions of bodies included in a given space are the same among themselves, whether that space is at rest, or moves uniformly forwards in a right line without any circular motion.

For the differences of the motions tending towards the same parts, and the sums of those that tend towards contrary parts, are, at first (by supposition), in both cases the same; and it is from those sums and differences that the collisions and impulses [*congressus et impetus*] do arise with which the bodies mutually impinge one upon another. Wherefore (by Law 2.) the effects of those collisions will be equal in both cases; and therefore the mutual motions of the bodies among themselves in the one case will remain equal to the mutual motions of the bodies among themselves in the other. A clear proof of which we have from the experiment of a ship; where all motions happen after the same manner, whether the ship is at rest, or is carried uniformly forwards in a right line.

We can interpret 'space' as what we would call today a (physical) coordinate system, or a reference frame. In the proof the initial data ("at first") are described in terms of position and velocity differences between bodies; since it is these differences which determine "collisions and impulses", if they are equal at the beginning, then the same "mutual motions of the bodies among themselves" will be observed.

Actually, this proof is far from satisfactory, and not just because not all logical links seem clear (is of all places Newton's *Principia* where it can be tacitly assumed that all physical interactions are derived from "collisions and impulses"?…), but because, surprisingly, the main hypothesis of the corollary, namely that the "space […] is at rest, or moves uniformly forwards in a right line without any circular motion", simply *plays no role in the proof*.

In fact the next corollary, which comes immediately after, reads as an afterthought. Here it is:

Corollary VI

If bodies, any how moved among themselves, are urged in the direction of parallel lines by equal accelerative forces; they will all continue to move among themselves, after the same manner as if they had been urged by no such forces.

For these forces acting equally (with respect to the quantities of the bodies to be moved), and in the direction of parallel lines, will (by Law 2.) move all the bodies equally (as to velocity), and therefore will never produce any change in the positions or motions of the bodies among themselves.

So equal acceleration on bodies, "any how moved among themselves", does not change how they "move among themselves". This corollary does not mention any including space, and yet the conclusion is framed in terms of mutual motions of bodies, just as in the previous corollary. In both results there is no direct reference to observers at rest in a "space".

We can first clarify this corollary by introducing a different set of coordinate changes:

$\mathbf{r}' = A\,(\mathbf{r} - t\,\mathbf{V} - (t^2/2)\,\mathbf{G}) + \mathbf{b}$, $t' = t + k$,

with $\mathbf{G}$ as a new vector constant. This set turns out to be also a group, of which the Galilei group is a subgroup (obtained by setting $\mathbf{G} = \mathbf{0}$). We can call it the *group of uniformly accelerated systems*. Now, it is clear that in a two-body system we have:

$$\mathbf{r}'_1 - \mathbf{r}'_2 = A\,(\mathbf{r}_1 - \mathbf{r}_2), \qquad (3)$$

and therefore it is true that in φ' the particles will "continue to move among themselves", i.e. with respect to one another (in particular preserving their distances), just as if there were no such «accelerative forces». Formula (3) can be considered to encapsulate the whole meaning of Corollary VI.

However, both statement and proof of Cor. VI would apply to a much more general group (an infinite-dimensional one):

$\mathbf{r}' = A\,(\mathbf{r} - \mathbf{h}\,(t)) + \mathbf{b}$, $t' = t + k$,

where $\mathbf{h}$: $R \rightarrow R^3$ is any differentiable function. Clearly even in this case (3) holds.

However, since $\mathbf{v}' = A(\mathbf{v} - \mathbf{V} - t\mathbf{G})$, we have also

$\mathbf{v}'_1 - \mathbf{v}'_2 = A\,(\mathbf{v}_1 - \mathbf{v}_2)$,

and therefore in the coordinate system φ' the law of motion of, say, m_1 is by (2)

$m_1\,\mathbf{a}'_1 = m_1\,A\,(\mathbf{a}_1 - \mathbf{G}) = A\,\mathbf{f}\,(\mathbf{r}_1 - \mathbf{r}_2, \mathbf{v}_1 - \mathbf{v}_2) - m_1\,A\mathbf{G}$

$= \mathbf{f}\,(\mathbf{r}'_1 - \mathbf{r}'_2, \mathbf{v}'_1 - \mathbf{v}'_2) - m_1\,A\mathbf{G}$,

which is *not the same law* verified by the inertial φ unless $\mathbf{G} = \mathbf{0}$. Thus, while it is true that all bodies "will all continue to move among themselves, after the same manner as if they had been urged by no such forces", this does not imply that the same laws of physics hold. In other words, we have *invariance of relative motions among bodies without Galilean relativity*. Newton's proof of Corollary V just misses this possibility.

In particular, while according to φ the centre of mass of a supposedly isolated two-body system moves with constant velocity, according to φ' the centre of mass has a *constantly accelerated* velocity.

But how can φ and φ' decide which one, between them, is observing a real and which one a fictitious force?

In Newton's view, the gold standard for this distinction is just how each of the two coordinate systems relates to absolute space. In the *Scholium* to the definitions of Book I, the author made a remarkable disclosure as to the reasons that induced him to write his treatise: he wrote it in order to provide criteria to distinguish between true from apparent motions. In other words, he meant to teach his readers how to detect, by physical means, their absolute motion! The original text of this important statement deserves to be reproduced (I introduced changes in the Motte-Cajori translation to be closer to the original text):

> Motus autem veros ex eorum causis, effectibus, & apparentibus differentiis colligere, & contra ex motibus seu veris seu apparentibus eorum causas & effectus, docebitur fusius in sequentibus. Hunc enim in finem tractatum sequentem composui.

«But how we are to obtain the true motions from their causes, effects, and apparent differences, and, conversely, from motions either true or apparent their causes and effects, shall be explained more at large in what follows. For to this end it was that I composed the following treatise.»

Lest this research programme should appear, in our different climate of ideas, unworthy of the great physicist, one must observe that, by saying so, Newton was claiming to be looking for a definitive solution to the most important scientific issue of his age: the Copernican controversy.

Hardly a humble task, especially four decades before James Bradley [7] had for the first time established on astronomical grounds that the Earth was moving with respect to the fixed stars.

However, when Newton comes to explain his methods to distinguish relative from absolute motions, he gives, as is well known, examples not for *any* accelerated motion, but only for *rotatory* motions (cf. his famous and much debated bucket experiment). No *physical* recipe is given on how to tell whether our laboratory is uniformly accelerating in the absolute sense. And Corollary VI suggests that our modern formulation of Galilean relativity might fail to capture the full strength of Newton's axiomatics, which includes, as we shall see, what Einstein will call, when developing general relativity, the *equivalence principle*.

4. Incommensurable theories of motion

The problem of identifying absolute rest in Newtonian physics surfaced during the celebrated bilingual Clarke-Leibniz correspondence. Criticizing Newton, Leibniz argued (my translation from the French text):

> For me, I pointed out more than once that I hold Space to be something purely relative, as Time; it is an order of Co-existences, as time is an order of successions. Since space indicates in terms of possibility an order of things which exist at the same time, insofar as they exist together, without dealing with their particular ways of existing: and when several things are seen together, one realizes that this is the order between them.

To this objection Clarke replied that, in case God decided to decelerate a uniformly moving universe, or to stop it altogether, observers would be able to realize whether they were, previously, uniformly moving or not:

> If Space was nothing but the Order of Things co-existing; it would follow, that if God should remove in a straight line the whole Material World Entire, with any swiftness whatsoever; yet it would still always continue in the same Place: and that nothing would receive any Shock upon the most sudden stopping of that Motion. And if Time was nothing but the Order of Successions of created things; it would follow, that if God had created the World Millions of Ages sooner than he did, yet it would not have been created at all the sooner.

Now, if space is purely relative, also motion is; therefore – as Leibniz remarked – there is no conceivable observation that would prove or disprove that the whole material universe is moving in absolute space: simply, those two states (uniform motion or rest with respect to absolute space) are "indiscernible", and as such there cannot be any "sufficient reason" for God to have decided in favour of one rather than of the other. It follows that they are strictly identical – even from God's viewpoint. The same argument holds for time.

In his reply, Leibniz appealed to two of his metaphysical principles which, combined, had a very damaging effect on the Newtonian foundations of natural philosophy: entities such as absolute space and absolute time just could not be admitted (not everybody was convinced, and in particular Euler published in 1750 a defense of absolute space and time [8]).

Incidentally, today every orthodox cosmologist would answer along Leibnizian lines to the question: 'Why did the Big Bang not occur sooner?', by stressing that 'before the Big Bang' makes no sense – time and the universe arose together.

Leibniz's and Newton's concepts of motion are an example of incommensurable cosmological theories (to use the term in Kuhn and Feyerabend's sense): theories in which the same statements may be true or false in one, and simply meaningless in the other. Paradoxically, between these authors it was the more metaphysical thinker, Leibniz, to advance the views that two centuries later the logical positivists, following in the footsteps of Ernst Mach and early Einstein, would recognize as more in agreement with their own opinions [9].

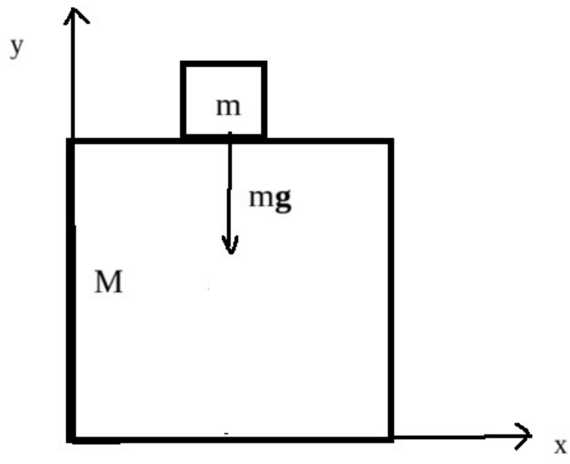
Notice that Corollary VI implies that the very same reply can be made as regards the possibility that, for instance (see above), the universe is uniformly accelerating. Obviously one cannot think of praying God, for the sake of settling a scientific dispute, to *stop* the universe in case it is uniformly moving, or to decelerate it to absolute uniform motion if it is uniformly accelerating. Thus, to ascertain whether the universe is absolutely moving or not should be considered as beyond human power – or, as Leibniz first held and the most unlikely of his followers later adopted, a meaningless question.

So how could Newton hope to honour the engagement solemnly taken at the moment of starting to compose his great work? Before dealing with this question it is convenient to make a somewhat pedagogical detour.

5. Forces in uniformly accelerating systems

One of the reasons a formal treatment of 'force' as the one sketched in section 2 is not often to be found in physics textbooks is, probably, that not all forces introduced in classical physics are *explicitly* interaction forces between particles. The clearest example is given by friction and medium resistance. These are forces which typically apply to special coordinate systems, with respect to which the relevant material surface or the relevant medium is at rest – no reduction of the friction or resistance to particle-to-particle interactions is attempted or even implied. As a matter of fact, a reduction to 'first principles' of the laws of friction has never been achieved, what has been called humorously "one of the dirty little secrets of physics" [10].

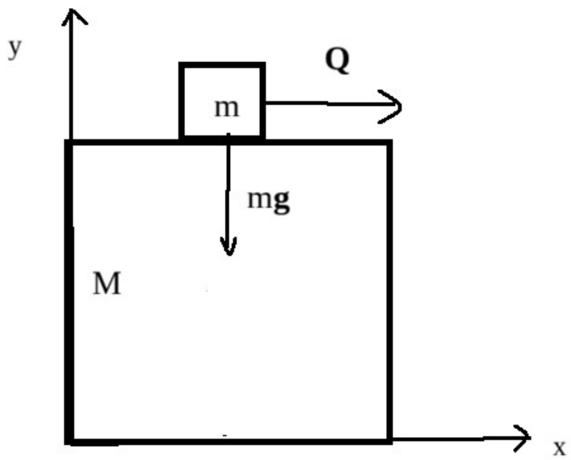
For instance, let us consider two rigid material cubes in an inertial system, a small cube of mass m on a large one of mass M, both subject to a constant gravity field. They are initially at rest with each other, and at this stage Corollary VI implies that nothing more would follow from the further information that they are at rest *in an inertial system*.



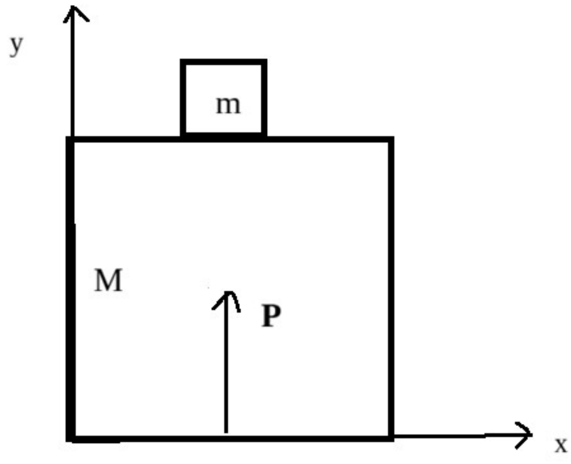
Suppose now that the small cube is subject to a force **Q** in the x-axis direction **i**; then a second force develops from the contact surfaces of the cubes, opposing **Q**: the static friction force $\mathbf{F}_s$, satisfying:

(3) $$\mathbf{F}_s = -\mathbf{Q} \quad \text{if } |\mathbf{Q}| \leq \mu_s mg,$$

where μ_s is the static friction coefficient. Clearly this force (which does not depend on the area of the contact surface) is hardly reducible to interaction forces between particles, and yet it can be said to obey Galilean relativity – since the same law would be used in any Galilean system.



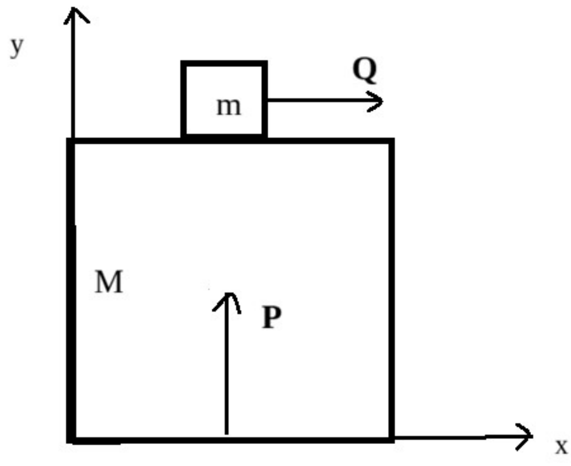
Now suppose that there is no gravity field but there is a constant force **P** from below:



This force gives the two-cube complex an acceleration **a** = **P**/(M+m). In the co-moving coordinate system the small cube acquires an apparent weight, which is:

- m**P**/(M+m)

This is similar to the situation illustrated by Einstein and Infeld with their elevator's example (anticipated as "chest" in Einstein's own previous popularization of relativity) in their popular conceptual history of physics [11].

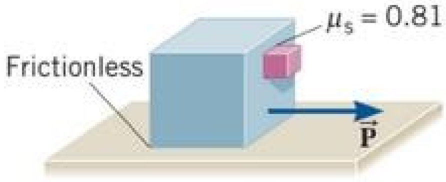


Can we use formula (3), including the value of the static friction coefficient, also in the uniformly accelerated coordinate system in case a force **Q** in direction **i** is impressed on the small cube? If we accept Einstein' equivalence principle, the answer is yes. And, as we have seen, Corollary VI may be read as containing it.

In physics textbooks there is some uncertainty on the role played by Galilei relativity. For instance in [12] there is no mention of relativity before arriving to the special theory of relativity. However there are also cases in which exercises are proposed where students are supposed to tacitly generalize relativity to uniformly accelerated systems.

Here is an interesting example taken from another popular college physics textbook [13].

71. **M** The drawing shows a large cube (mass = 35 kg) being accelerated across a horizontal frictionless surface by a horizontal force $\vec{\mathbf{P}}$. A small cube (mass = 6.0 kg) is in contact with the front surface of the large cube and will slide downward unless $\vec{\mathbf{P}}$ is sufficiently large. The coefficient of static friction between the cubes is 0.81. What is the smallest magnitude that $\vec{\mathbf{P}}$ can have in order to keep the small cube from sliding downward?



PROBLEM 71

In this exercise, one is implicitly asked to generalize the relativity principle to uniformly accelerating systems, by treating the large cube, which is sliding with a constant acceleration **a** (to be computed from the information given) on a frictionless horizontal plane surface, as equivalent to an inertial system with a 'gravity acceleration' equal to -**a**, where

$$(M+m)\mathbf{a} = \mathbf{P}\ .$$

Thus the condition guaranteeing that the small cube will not slide vertically is:

$$mg \leq \mu_s\, m\,|\mathbf{P}|/(M+m)$$

and the minimum value of $|\mathbf{P}|$ compatible with the no-sliding requirement is therefore $(M+m)g/\mu_s$.

What is remarkable in this exercise is that we are asked to apply the static friction formula (3) (including the value of the friction coefficient) in a uniformly accelerated system, just as we had an inertial system, and what is here the constant *horizontal* acceleration we could re-interpret as *vertical* – that is, as 'weight'. This is clearly quite a move from Galilean relativity, but it is consistent with Newton's Corollary VI.

The only author, to my knowledge, to have remarked that Newton's mechanics countenances far more than Galilean relativity was the eminent astrophysicist S. Chandrasekhar, just after the statement of Corollary VI (and not in the section "The Newtonian Principle of Relativity"!) in his book commenting on Newton's *Principia* [14]. There he writes, sparingly: "Notice that this corollary shows that Newton's laws hold even in *some accelerated frames*." (italics added), but he does not elaborate on this very interesting point.

6. The architecture of the universe

Without any hypotheses on the general structure of the world it is impossible to make advances on the issue of the relationship of the material universe with absolute space. In a sense we can say that in the *Principia*, before reaching Book III, the reader may well doubt as to the exact relevance of the mathematical treatment in most of the previous two books.

For instance, suppose that in the Newtonian universe there was an infinite (or very large) material wall W with a constant mass surface density ρ, like in the following figure, representing the ordinary material universe as all contained within a small ellipsoid S. Incidentally, it is worth remembering that, notwithstanding the ordinary simplifying assumptions of isotropy and homogeneity leading to the standard relativistic cosmological space-times, observational astronomy has detected in our universe large structures, which are inconsistent with these properties, three of them having been named as "great walls" [15].

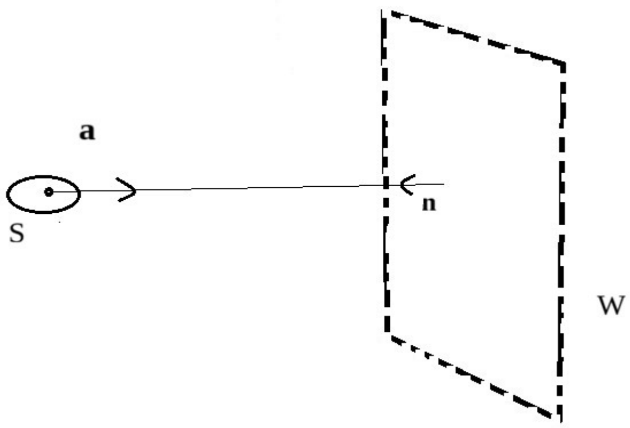


Figure 1: An infinite material wall in the universe.

We can suppose that by assuming a sufficiently large mass difference between S and W, $m_S << m_W$, wall W would not be significantly influenced by the gravitational attraction of S, and therefore could be considered, for all practical purposes, to be absolutely at rest.

On the other hand, as explained in standard physics textbooks (often when treating not gravitation, but the electric field of a capacitor – a problem which is formally close to this in the approximation required), an infinite material plan with a constant mass density attracts any particle with the same gravitational field directed orthogonally to the plan – independently of the distance from the particle to the wall:

$\mathbf{G} = - 4\pi G\rho\ \mathbf{n}$

Thus, if such a material wall belonged to the furniture of the universe, all particles would be absolutely accelerating with the same constant acceleration **a = G**, but the relative motions among them would be just the same as if no such wall existed (Corollary VI).

If we suppose W to be so distant from S that no radiation would ever come from it to S so long as mankind will survive, then no physical evidence will ever be available to us making it likely that any such material structure existed.

This argument depends on two assumptions perfectly consistent with the *Principia*'s axioms:

1) the concept that gravitation is transmitted instantaneously, while light (in the visible or invisible spectrum) has a finite speed;

2) inertial mass and gravitational mass are equivalent, and can be functionally and numerically identified.

The second assumption is essential to the validity of Einstein's equivalence principle, and reaches back to Galileo's insistence on the equal acceleration of falling bodies, whatever their inner constitution and weight – if all medium's resistance can be ignored (see the experimental evidence based on the use of an air pump in *Principia*, III, after Proposition 10).

To historians interested in heuristics, I suggest that Einstein might have been led to his equivalence principle by noticing the resemblance and editorial proximity (pp. 20-21 in the first 1687 edition of *Principia*) between corollaries V and VI. As we have seen, while Corollary V expresses the principle of Galilean relativity, Corollary VI goes further, and is closer (although not identical) to Leibniz's concept of *motion as intrinsically relational* – what can be called, following Poincaré, the *principle of relative motion* [16]. The two notions have often been blurred into a single one – a conceptual mistake which has been for a century the source of many misunderstandings among both supporters and critics of relativity. A signal example of this occurred in the very introductory section of Einstein's 1905 electrodynamics paper [17] (for a discussion see [18]).

I am not aware that this link has ever been noticed by historians, but I hope to be forgiven if I missed it in the huge literature on the birth of general relativity. I surmise, however, that a widespread neglect of these textual data might be associated, in turn, to Cohen's silence on the principle of relativity in his extensive commentary.

7. Newton's cosmological solutions

In the book III of Principia, "On the system of the World", Newton tackled, at last, the issue of fixing a reference place that can be considered to be absolutely at rest.

Hypothesis I

That the centre [centrum] *of the system of the world is immovable.*

This is acknowledged by all, while some contend that the earth others that the sun, is fixed in that centre. Let us see what may from hence follow.

Proposition 11. Theorem 11

That the common centre of gravity [centrum gravitatis] *of the earth, the sun, and the planets, is immovable.*

For (by Cor. IV of the Laws) that centre either is ar rest, or moves uniformly forwards in a right line; but if that centre moved, the centre of the world would move also, against the Hypothesis.

In the reproduced passage we see Newton's crucial move: he postulates ("Hypothesis I") that the universe is symmetrically spherical – otherwise the notion of its having a "centre" (here he does not write 'centre of gravity'), however identified, would make no sense. How does he justify this assumption? By referring to general consensus – both Ptolemy and Copernicus agreed... well, they 'just' differed on whether the centre was occupied by the Earth or the Sun! (For Newton, it is neither).

The consequence derived from Hypothesis I is that the centre of gravity of the solar system must be absolutely at rest, the implicit assumption being that it is at rest with respect to the centre of the universe. How do we know that the latter condition holds? Let us see Corollary IV:

Corollary IV

The common centre of gravity of two or more bodies does not alter its state of motion or rest by the actions of the bodies among themselves; and therefore the common centre of gravity of all bodies acting upon each other (excluding external actions and impediments) is either at rest, or moves uniformly in a right line.

This can be seen as a generalization of the Law of Inertia ("Lex I"), where a single isolated body is substituted by the centre of gravity of an isolated system of bodies, that is, a system of bodies on which the resultant of the external forces on it vanishes. But how do we know that this cosmological coincidence – the vanishing of such resultant force – holds?

The easiest way is to assume that, outside of the solar system, the distribution of distant matter can be modelled as a spherical shell with constant (or constant for each spherical slice) mass density. In fact, as is well known, the gravitational field created inside a material sphere by a constant mass density on the sphere vanishes.

Therefore the possibility of distinguishing between a universe as in Figure 1 and a universe as in Figure 2 *depends on cosmological hypotheses*, not on observable local physics, including observable violations of the principle of Galilean relativity.

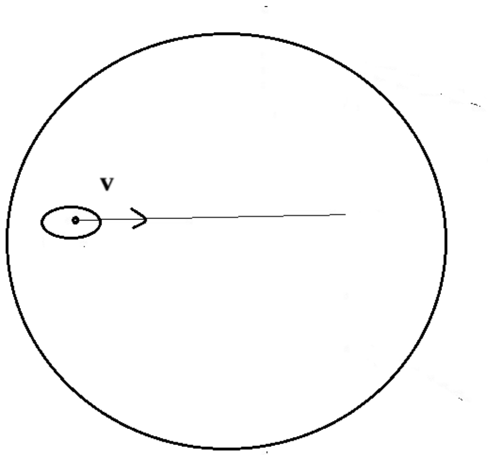


Figure 2: The solar system (...or the Galaxy) inside a material hollow sphere.

It is worth mentioning in this context that the very simple proof of Corollary IV, given by Newton, based on the Third Law of Motion fails, in fact, for some material systems with infinitely many components bound by gravitation only – thus we find even at this basic stage Newton's dynamics makes hidden (if plausible) assumptions on what physically can exist [19].

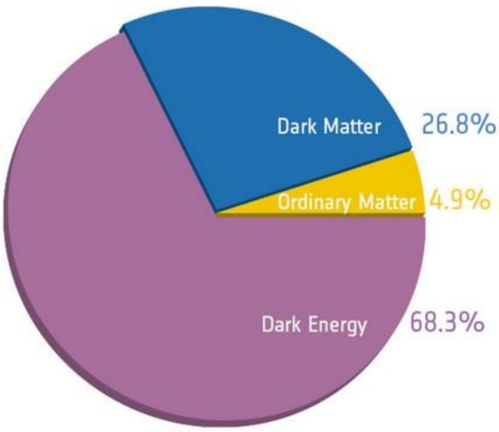
In 1692, just five years after the first edition of the *Principia*, Newton had an epistolary exchange [20] with a classical scholar, Richard Bentley, who was preparing his cycle of Boyle lectures on natural theology, and – understandably – wanted to refer to the best contemporary opinion on the structure of the universe. In his response, Newton made room for the possibility that the material universe could be infinite, contrasting it with the alternative of a finite universe, in which

> "the Matter on the outside of this Space, would by its Gravity tend towards all the Matter on the inside, and by consequence fall down into the middle of one great spherical Mass",

while this would not happen "if the Matter was evenly disposed throughout an infinite Space". Today we should say that, because of Sundman's inequality, a total collapse of a finite universe is not unavoidable, but would require, under regularity conditions, a vanishing total angular momentum (see for instance [21, p. 59]).

In any case, a single material 'island universe' in infinite space would have elicited the kinds of objections advanced by Leibniz.

Perhaps today these issues seem very far from the elevated standpoint which modern cosmology is supposed to have reached. However, this popular presumption is largely unwarranted. It is true that general relativity has succeeded in establishing a link between the material content of the universe and its (changing) geometry, but as to what is contained in the universe we are still largely left in the dark. And here 'largely' can be taken literally. The following is a figure showing how the material content of the universe is today divided between ordinary matter, dark matter, and dark energy:



The dark energy is a free parameter in Einstein's field equation – the cosmological constant first introduced by Einstein in 1917, and abandoned in 1931 [22, 23].

This is not the place to enter into details, but surely this partitioning is enough to suggest that any announcement that the cosmological problem has been solved since the introduction of general relativity must be considered as greatly exaggerated.

8. By way of conclusion

We have seen that Galilean relativity in Newtonian physics plays an ambiguous role, being besieged by a much more substantial statement on relative motion; this often neglected circumstance reflects both in the often marginalized treatment of Galilean relativity in textbooks (an exception is the classic [4]) and in historical accounts of Newtonian mechanics that just seem to find hard to make adequate room for it. And yet, as we have seen (section 5), even undergraduate exercises require, for a full solution, a rather sophisticated understanding of its place in classical physics.

The problems raised in the Leibniz-Clarke controversy have often been represented as finally settled by Einstein's general relativity. Indeed, the Copernican controversy has been described by Einstein and Infeld [11, p. 224] and other eminent authors as dissolved, insofar as it was supposed to be based on conceptual misunderstandings eventually disposed of by general relativity (cf. [22] for a critical assessment of this claim). In this paper we have seen that the issue of the relational vs absolute nature of motion cannot be solved by pure theoretical reflection if we want to transform it into a genuinely physical issue. For that, there is need of substantial cosmological postulates linking local physics to the global structure of the universe. To put it in other terms, if we assume not to know anything on the material content of the universe, then in the Newtonian framework local physics is unable to distinguish absolute rest not just from absolute uniform motion, but also from some kinds of absolute accelerated motion, including the uniformly accelerated one.

This suggests that a discussion such as the one contained in sections 6-7 of this paper should find its way even into undergraduate courses in physics, contrary to a virtual absence of cosmology in institutional physics courses – at all levels. Probably one of the reasons for this absence is the authors' unstated belief that cosmology, far from being needed as a foundation of physics, is the crowning achievement of modern physics, and as such its presentation may well be delayed to later stages of one's formal education… stages, perhaps, never to be reached.

It is arguable, on one hand, that cosmological models, even in the form of 'toy' models (i.e. clearly unrealistic) are better than no models at all and, what is more, that they are necessary, as we have seen, to build a consistent concept of physical law – even within classical mechanics.

On the other hand, it should be admitted that the state of contemporary cosmology is not so satisfactory, let alone final, contrary to the embellished portrait that it is often offered in journalistic and popular accounts. This fact should be openly communicated to students, not least to convey to them a correct appreciation of the difficulty of the problems that such giants of natural philosophy as Galileo, Descartes, Leibniz, and Newton had to face.

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