Enhancement of omnidirectional bandgap in graphene based quasi-periodic one dimensional photonic crystal heterostructures

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Abstract: We study the omnidirectional reflection (ODR) in one dimensional photonic crystal (PC) structures consisting of alternate layers of graphene as material of high refractive index and SiO\textsubscript{2} as material of low refractive index. The effects of the thickness of layers and incidence angles on the spectral reflectance have been investigated using transfer matrix method (TMM). The proposed structure gives high reflection within a wide range of wavelengths in the visible and near infrared region and can be used effectively in wavelength filters, optical resonators and mirrors. We also propose here the considerable enhancement of omnidirectional reflection band in near infrared region by changing the design parameters using a gradual constant. Due to considerable control of absorption and small dissipation of electromagnetic energy in graphene, the damping and skin effect in the PC are also obscured.

Keywords: Omnidirectional reflection, Photonic crystal, Graphene

1. Introduction

Photonic Crystals (PCs) are optical nanostructures with a periodic modulation in the refractive index on the length scale comparable to optical wavelength. They are characterized by electromagnetic forbidden bands or photonic band gaps (PBGs). In other words, the propagation of electromagnetic waves, whose frequencies lie within the PBGs, is prohibited. This unique feature of the photonic crystal structures controls dramatically the flow of light within the structure and can lead to many potential applications in field of photonics [1–3]. The propagation of photons in PCs is analogous to propagation of electrons in semiconductor crystals where the effect of periodic refractive index in PCs is same as the effect of periodic potential function on propagation of electrons in semiconductors [4,5].
In these structures the refractive index is a periodic function in space and if the refractive index is periodic only in one dimension then the structure is called one dimensional photonic crystal (1DPC), if it is periodic in two dimensions and three dimensions then the structure is known as two dimensional photonic crystal (2DPC) and three dimensional photonic crystal (3DPC) respectively [6].

PCs that work in microwave and far-infrared regions are relatively easier to fabricate. However, PCs that work in visible and the infrared (IR) regions, especially, 3D PC are difficult to fabricate because of their small lattice constants, which have to be comparable to the wavelength. Therefore, 1D PCs, which can easily be produced by the thin film deposition techniques, are preferable for use in the visible and IR regions. The simplest 1DPC is an alternating stack of two different mediums having reflection properties which find them used in variety of applications including high efficiency mirrors, Fabry Perot cavities, optical filters and feedback lasers [7-12]. In recent years, omnidirectional reflectors i.e. nearly 100% reflectivity irrespective of angle of incidence and state of polarization are one of the most widely used optical devices and a great deal of work has been done on these reflectors [13–29].

In metallic reflector, light can be reflected over a wide range of frequencies for arbitrary incident angles however, at higher frequencies considerable amount of power is lost due to the absorption. In comparison to metallic reflectors a multilayer dielectric reflectors have high reflectivity in a certain range of frequencies, but the reflectivity is very sensitive to the incident angles [30-39]. The range of reflected frequency of multilayer dielectric reflector can be enhanced by the appropriate selection of the material parameters and layer thickness [40-48].

The PBG can be enlarged by arranging the PCs with asymmetric topology structures [49] and introducing meta-materials [18]. Recently the PBGs are enhanced in 1DPC with Fibonacci and Thue-Morse quasi periodic structure [50-51]. The plasma PCs with fractal structures can also enlarge the bandwidth of PBGs [52-60].

Graphene is a gapless semiconductor with mass less electrons and holes. The 2D electron system was experimentally observed in graphene which is a 2D honeycomb lattice of carbon atoms [61-64]. Graphene has unique optical properties due to its structure. In infrared region the transparency of graphene is defined by fine structure constant [65-67]. Graphene based PCs can be used in the infrared region of spectrum, over wide range of temperatures and the PBG can controlled by changing the thickness of the dielectric layers between the graphene layers and by doping [68-70]. The size of graphene based PC can be larger than size of metallic PC due to small dissipation of electromagnetic wave energy.

In this paper we propose the omnidirectional band in the multilayered 1D PC structure consisting alternate layers of graphene and SiO₂. The theoretical analysis is based on the transfer matrix method [2]. Here we have used 5 stacks of 5 alternate layers of Graphene as high refractive index and SiO₂ as low refractive index. We also propose here the considerable enhancement of omnidirectional reflection band in near infrared region by changing the design parameters using a
gradual constant. We observe that graphene based photonic crystal structure can be used as a good candidate for complete inhibition of transmission of frequency in near infrared region. The proposed structure can be used as wavelength filter or broad reflector in the visible and near infrared spectrum which is very useful in many imaging sensors in the field of optical technology.

2. Theoretical analysis

To calculate the dispersion relation and reflection characteristics for the incident electromagnetic wave, the Maxwell’s equations are solved numerically by the transfer matrix method [2].

![Fig 1. Periodic refractive index profile of the structure having refractive indices $n_1$ and $n_2$ respectively](image)

The geometry of the structure under study is shown in the Fig. 1. Considering the propagation of EM wave along x-axis normal to the interface in one-dimensional system composed of periodic arrays of two different materials with a refractive index $n_1$ and $n_2$ and layer thickness $d_1$ and $d_2$. The indices of refraction of the system are given as,

$$n(x) = \begin{cases} n_1, & 0 < x < d_1 \\ n_2, & d_1 < x < d_2 \end{cases}$$

with $n(x) = n(x + d)$. Where $d = d_1 + d_2$ is the period of the structure. The electromagnetic field distribution within each layer can be expressed as the sum of right- and left-hand side propagating wave. The electric field within the both layers of the $n^{th}$ unit cell can be written as,

$$E_1(x) = \left[(a_n e^{-i k_1 (x-n d)}) + (b_n e^{i k_1 (x-n d)})\right] e^{i \omega t}$$

$$E_2(x) = \left[(c_n e^{-i k_2 (x-n d)}) + (d_n e^{i k_2 (x-n d)})\right] e^{i \omega t}$$

Where $k_i = \left[\left(\frac{n_{i \omega}}{c}\right)^2 - \beta^2\right]^{\frac{1}{2}} = \frac{n_{i \omega}}{c} \cos \theta_i , \ (i = 1, 2)$

$\theta_i$ is the ray angle in the $i^{th}$ layer, $\beta$ is the propagation constant and $n_i$ is the refractive index of the constituent layers. The coefficients $a_n, b_n, c_n,$ and $d_n$ are related through the continuity boundary
conditions at the interfaces \( x = (n-1)d \) and \( x = (n-1)d + d \). This continuity condition leads to the matrix equations, which relates the coefficient in the first layer of the \( n \)th cell, given as

\[
\begin{pmatrix}
a_{n-1} \\
\hline
b_{n-1}
\end{pmatrix} = T_n \begin{pmatrix}
a_n \\
\hline
b_n
\end{pmatrix}
\]  

(4)

where \( T_n \) is called the transfer matrix given by

\[
T_n = \begin{pmatrix} A & B \\ C & D \end{pmatrix}
\]  

(5)

The matrix elements \( A, B, C \) and \( D \) are

\[
A = e^{ik_1d_1} \left[ \cos k_2d_2 + \frac{1}{2} i \left( \eta + \frac{1}{\eta} \right) \sin k_2d_2 \right] 
\]  

(6)

\[
B = e^{-ik_1d_1} \left[ \frac{1}{2} i \left( \eta - \frac{1}{\eta} \right) \sin k_2d_2 \right] 
\]  

(7)

\[
C = e^{ik_1d_1} \left[ - \frac{1}{2} i \left( \eta - \frac{1}{\eta} \right) \sin k_2d_2 \right] 
\]  

(8)

\[
D = e^{-ik_1d_1} \left[ \cos k_2d_2 - \frac{1}{2} i \left( \eta + \frac{1}{\eta} \right) \sin k_2d_2 \right] 
\]  

(9)

The parameter \( \eta \) depends on the polarization. For the TE and TM polarizations, \( \eta \) is given by

\[
\eta_{TE} = \frac{k_1}{k_2} 
\]  

(10)

and

\[
\eta_{TM} = \frac{k_1n_2^2}{k_2n_1^2} 
\]  

(11)

For finite stacks, the coefficient of right and left hand side propagating wave in both sides of the multilayer structure \( a_N \) and \( b_N \), are calculated by multiplying transfer matrix of each cell as

\[
\begin{pmatrix}
a_0 \\
\hline
b_0
\end{pmatrix} = T_1 \cdot T_2 \cdots T_N \begin{pmatrix}
a_N \\
\hline
b_N
\end{pmatrix}
\]  

(12)

where \( N \) is the total number of the cell. The coefficient of reflection is given by solving above matrix equation with the condition \( b_N = 0 \) as

\[
r_N = \frac{b_0}{a_0} 
\]  

(13)

Thus the reflectivity (or reflectance) of the structure may be calculated as

\[
R_N = |r_N|^2 
\]  

(14)

Now, according to Bloch theorem, the electric field vector is of the form \( E(x) = E_{K(x)} e^{i(\omega t - Kx)} \) where \( E_{K(x)} \) is periodic with the ‘d’. For the determination of \( K \) as a function of eigenvalue, the equation is written as

\[
\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} = e^{iKd} \begin{pmatrix} a_n \\ b_n \end{pmatrix} 
\]  

(15)

The solution of this matrix equation leads to the dispersion relation for the PC structure

\[
K(\omega) = \left( \frac{1}{d} \right) \cos^{-1} \left[ \cos(k_1d_1) \cos(k_2d_2) - \frac{1}{2} \left( \eta + \frac{1}{\eta} \right) \sin(k_1d_1) \sin(k_2d_2) \right] 
\]  

(16)
3. Results and discussion

Here we have considered the multilayered 5 stacks of 5 alternate layers of graphene and SiO₂. The structure is described by Fig. 2 by the following sequence -

\[(AB)^5/(A_1B_1)^5/(A_2B_2)^5/(A_3B_3)^5/(A_4B_4)^5\]

The refractive index of graphene is taken as \(n_1 = 2.7\) and that of SiO₂ is taken as \(n_2 = 1.46\) in the wavelength range from 400 nm to 2000 nm. The central wavelength for computation is taken as \(\lambda° = 1200\) nm. According to quarter wave stack condition, in the stack (AB) the thickness of graphene layer is \(a = \frac{\lambda°}{4n_1}\) = 111.11 nm and the thickness of SiO₂ is \(b = \frac{\lambda°}{4n_2}\) = 205.48 nm. Now we have introduced a gradual multiplication constant \(\gamma\) such that in stack \((A_1B_1)\) the thickness of graphene layer is \(a_1 = \gamma a\) and the thickness of SiO₂ is \(b_1 = \gamma b\). In the same way in stack \((A_2B_2)\) the thickness of graphene layer is \(a_2 = \gamma a_1\) and the thickness of SiO₂ is \(b_2 = \gamma b_1\). Similarly in stack \((A_3B_3)\) the thickness of graphene layer is \(a_3 = \gamma a_2\) and that of SiO₂ is \(b_3 = \gamma b_2\) also in stack \((A_4B_4)\) the thickness of graphene layer is \(a_4 = \gamma a_3\) and that of SiO₂ is \(b_4 = \gamma b_3\). The edges of PBG shift towards higher frequencies or lower wavelength when we increase the angle of incidence and at Brewster’s angle TM mode will not be reflected and we will not get a complete PBG but it does not mean that there will no OBG, the incident wave from outside cannot couple to Brewster window if the maximum refracted angle from outside medium air (\(n_0\)) is less than the Brewster angle for the photonic structure and this will result in reflection of TM modes as well which will give the reflection at all angle of incidence. At the air and \(n_1\) interface, by Snell’s law \(n_0 \sin \theta_0 = n_1 \sin \theta_1\) so maximum refracted angle will be \(\theta_1^{\text{max}} = \sin^{-1}(n_0/n_1) = \sin^{-1}(1/2.7) = 21.74°\).

Also from Brewster’s law at \(n_1\) and \(n_2\) interface \(\theta_B = \tan^{-1}(n_2/n_1) = \tan^{-1}(1.46 / 2.7) = 28.4°.\) since \((\theta_B > \theta_1^{\text{max}})\) condition is satisfied by our design parameters so the incident wave from the ambient medium will not coupled in the Brewster window, and we will still get omnidirectional reflection.
band. The difference between the lower bandgap edge of TE polarized waves at normal incidence and the upper bandgap edge of the TM polarized waves at perpendicular incidence is called the omnidirectional bandgap (OBG).

For a complete periodic structure we take $\gamma = 1$ in our proposed structure and Fig. 3 describes the normalized wave vector for both TE waves at $\theta = 0^\circ$ and TM waves at $\theta = 89^\circ$ and the OBG is also shown. The upper and lower edges for omnidirectional band gap is found to be $\lambda_U = 1125$ nm and $\lambda_L = 1006$ nm so the OBG is 119 nm which is 7.4% of the total considered wavelength range ($\lambda_C$). The same result is shown by the reflectivity diagram for TE modes at $\theta = 0^\circ$ and TM modes at $\theta = 89^\circ$ in Fig. 4. Now we change the design parameter by considering $\gamma = 1.1$, the reflectivity diagram and omnidirectional reflection band is shown in Fig. 5. The upper and lower edges for OBG is found to be $\lambda_U = 1561$ nm and $\lambda_L = 1011$ nm so the OBG is 550 nm which is 34.3% of the total considered wavelength range. Here we find a considerable increase in omnidirectional reflection band. We can further increase this band by taking $\gamma = 1.15$, Fig. 6 shows the reflectivity diagram and OBG. Here the upper and lower edges for omnidirectional band gap is found to be $\lambda_U = 1807$ nm and $\lambda_L = 1014$ nm so the OBG is 793 nm which is 49.5% of the total considered wavelength range. To further increase the OBG we consider $\gamma = 1.2$, Fig. 7 shows the reflectivity diagram and OBG. The upper and lower edges for OBG is found to be $\lambda_U = 2086$ nm and $\lambda_L = 1037$ nm so the OBG is 1049 nm which is 65% of the total considered wavelength range. The summary of the theoretical observations are described in table 1.

We have investigated theoretically the OBG in proposed 1DPC using graphene films. It is found that by changing the design parameters of the proposed photonic structure by a gradual constant there is considerable increase in the OBG. Hence the proposed structure can be used as a broad reflector in the visible and near infrared frequencies and can be tuned according to the design parameters. These types of optical filters may have potential applications in optical technology and optical communication.
Table 1: Enhancement in OBG using different values of gradual constant ($\gamma$)

<table>
<thead>
<tr>
<th>S.No</th>
<th>Gradual constant ($\gamma$)</th>
<th>Lower band edge of OBG ($\lambda_L$) (for $\theta=0^\circ$, TE)</th>
<th>Upper band edge of OBG ($\lambda_U$) (for $\theta=89^\circ$, TM)</th>
<th>OBG ($\Delta\lambda_{OB}$)</th>
<th>$\Delta\lambda_{OB}/\lambda_c$%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1006 nm</td>
<td>1125 nm</td>
<td>119 nm</td>
<td>7.4%</td>
</tr>
<tr>
<td>2</td>
<td>1.1</td>
<td>1011 nm</td>
<td>1561 nm</td>
<td>550 nm</td>
<td>34.3%</td>
</tr>
<tr>
<td>3</td>
<td>1.15</td>
<td>1014 nm</td>
<td>1807 nm</td>
<td>793 nm</td>
<td>49.5%</td>
</tr>
<tr>
<td>4</td>
<td>1.2</td>
<td>1037 nm</td>
<td>2086 nm</td>
<td>1049 nm</td>
<td>65%</td>
</tr>
</tbody>
</table>

Fig 3: Normalized wave vector in Graphene-SiO$_2$ proposed structure for $\gamma=1$, for TE, $\theta=0^\circ$ and TM, $\theta=89^\circ$ modes also showing the OBG lower edge at $\Lambda/\lambda=0.2814$, and upper edge at $\Lambda/\lambda=0.3146$. Since $\Lambda=a+b=316.59$ nm, $\lambda_L=(316.59/0.3146)=1006$ nm, $\lambda_U=(316.59/0.2814)=1125$ nm.
Fig. 4: Reflectivity variation with wavelength describing the Omnidirectional reflection band (ODR) for Graphene-SiO$_2$ proposed structure for $\gamma=1$, for TE, $\theta=0^\circ$ and TM, $\theta=89^\circ$ modes, showing $\lambda_L=1006$ nm, $\lambda_U=1125$ nm.

Fig. 5: Reflectivity variation with wavelength describing the Omnidirectional reflection band (ODR) for Graphene-SiO$_2$ proposed structure for $\gamma=1.1$, for TE, $\theta=0^\circ$ and TM, $\theta=89^\circ$ modes, showing $\lambda_L=1011$ nm, $\lambda_U=1561$ nm.
Fig. 6: Reflectivity variation with wavelength describing the Omnidirectional reflection band (ODR) for Graphene-SiO$_2$ proposed structure for $\gamma=1.15$, for TE, $\theta=0^\circ$ and TM, $\theta=89^\circ$ modes, showing $\lambda_L=1014$ nm, $\lambda_U=1807$ nm.

Fig. 7: Reflectivity variation with wavelength describing the Omnidirectional reflection band (ODR) for Graphene-SiO$_2$ proposed structure for $\gamma=1.2$, for TE, $\theta=0^\circ$ and TM, $\theta=89^\circ$ modes, showing $\lambda_L=1037$ nm, $\lambda_U=2086$ nm.
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