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Anisotropic bulk viscous string cosmological models with heat flux

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Abstract: Bulk viscosity coupled with cosmic string for Bianchi type VI0 cosmological model is investigated in the context of Self creation theory of gravitation. Field equations are solved for two different cases by considering relation between metric potentials. It is assumed that scalar function is proportional to the scale factor. It is found that the universe is anisotropic and stable. The early universe is highly heated. Some physical parameters are also discussed in details.

Keywords: Bulk Viscosity, String, Heat, self creation theory

1. Introduction

Cosmology is the study of the large and small structures of the universe. It draws on knowledge from other science, such as physics and astronomy and assembles a physically all inclusive cosmic picture [1]. In recent years, there has been a considerable interest in cosmological models with the help of theories of gravitation. Einstein's general theory of relativity has been successful in finding various models of the universe. However, general relativity does not account satisfactory for inertial properties of matter. It led to foundation of alternative theories of gravitation. Scalar tensor theories of gravitation are viable alternative to general relativity due to their cosmological applications. They are considered to be essential to describe the gravitational interactions near the plank scale; string theory, extended inflation and many higher order theories [2] imply scalar field. They are based on introduction of scalar field besides the metric tensor field. Brans-Dicke [3], Nordtvedt [4], Saez Ballester [5], Self creation [6] are important scalar tensor theories. Barber [6] proposed two self creation theories of gravitation. The first is a modification of Brans-Dicke theory. Brans [7] has pointed out that this first theory is not only in disagreement with experiment but is actually inconsistent, in general. Later, Barber introduced second theory which is modification of general relativity. In this theory, the scalar field acts as reciprocal to the gravitational constant. It divides the matter tensor [8]. An axially symmetric space time in the presence of perfect fluid in Barber second self creation theory is studied by Reddy and Naidu [9]. Katore and Shaikh[10] studied Einstein – Rosen string cosmological models in Barbers second self creation theory.

Cosmic strings play an important role in the early stage of evolution of the universe. Cosmic strings may be created during phase transitions [11]. Strings are one dimensional topological defect. They act as a source of gravitational field [12]. The gravitational effect of cosmic strings has been extensively discussed by Vilenkin [13], Gott [14], Venkateswarlu [15], Katore [16] and Singh [17]. Katore and Hatkar [18] have studied hypersurface homogeneous cosmological models for string using linearly varying deceleration parameter.

Cosmological model with bulk viscosity are also important since large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation suggest that matter behave like viscous fluid at early stage [19]. It is associated with the GUT phase transition and string creation. The viscosity theory of relativistic fluids was first suggested by Eckart [20] and Landau and Lifshitz [21].Bulk viscosity is the simplest way to study entropy in cosmology. It arises any time a fluid expands rapidly and ceases to be in thermodynamic equilibrium. Misner [22] studied the effect of viscosity on the evolution in the cosmological models. Singh [23] investigated the effect of bulk viscosity on the early evolution of universe for a spatially homogeneous and isotropic FRW model. Sancheti *et al*. [24] obtained solutions of the field equations in the presence of viscous fluid with the aid of hypersurface homogeneous space time in the context of $f(R)$ theory of gravitation. Exact solutions of the isotropic homogeneous model with bulk viscosity being a power function of energy density are obtained by Santos *et al.* [25]. Singh *et al.* [26] studied anisotropic Bianchi type II viscous fluid models with time dependent gravitational and cosmological constant.

In this paper, we obtain Bianchi type VI_0 bulk viscous sting cosmological model together with heat flux in Barber’s second self creation theory of gravitation. The plan of the paper is as follows. We present the metric and field equations in sections 2. Sections 3 deal with solution of the field equations. The last section 4 contains some conclusion.

2. Metric and field equations

Bianchi type space times play a vital role in understanding and description of the early stages of evolution of the universe [27]. We consider the spatially homogeneous and isotropic Bianchi type VI_0 metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2hx}dy^2 + C^2e^{2hx}dz^2, \tag{1}$$

where A,B,C are function of cosmic time t and h is a non zero constant. Bianchi type VI0 massive string cosmological models with magnetic field are investigated by Pradhan and Bali [28]. Exact solutions of spatially homogeneous anisotropic Bianchi type VI_0 with two co-moving perfect fluid as source are obtained by Coley and Dunn [29]. Katore and Sancheti [30] studied magnetized anisotropic dark energy with constant deceleration parameter. Ram and Singh [31] presented an exact solution of the vacuum Brans Dicke field equations for cosmological model of Bianchi type VI_0. Katore *et al.*[32] have studied Bianchi type VI0 cosmological models in presence of modified chaplygin gas in the context of general relativity.

The field equations in Barber's second self creation theory of gravitation are

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi\phi^{-1} T_{ij} \tag{2}$$

and scalar field is defined as

$$\phi = \phi^{k}_{;k} = \frac{8\pi}{3} \mu T , \tag{3}$$

where T is the trace of the energy momentum tensor that describes all non gravitational and non scalar field theory. μ is a coupling constant to be determined from the experiment $|\mu| < 10^{-1}$. When $\mu \to 0$, the theory approaches the general relativity in every respect. Recently, Banerjee *et al.*[33] presented an irrotational Bianchi type V model under the influence of shear and bulk viscosity together with heat flow. Tikekar [34] obtained non singular solutions with heat flux. Ram *et al.*[35] discussed the field equations in Saez Ballester scalar tensor theory of gravitation for a Bianchi type V model filled with viscous fluid together with heat flow. Katore *et al.*[36] have discussed string cosmology in modified *f(R)* theory of gravitation.

The early universe would be very different with different kind of physical properties than the present day universe. We need to consider all possibilities of matter distribution. We shall deal with some details of these physical distributions which we believe played a crucial role in the structure formation of the universe. Our intension is to analyze nature of probability of matter considered in this paper. It will be interesting to see whether this is merely an artificial combination or yield some meaningful results. Here, the energy momentum tensor for bulk viscous string cloud with heat flow is given by

$$T_{ij} = \left(\rho + \overline{P}\right) u_i u_j + \overline{P} g_{ij} + s_i u_j + s_j u_i - \lambda x_i x_j , \tag{4}$$

$$\text{where } \overline{P} = P - \varsigma u^{i}_{;i} , \ \rho = \rho_p + \lambda , \tag{5}$$

here ρ is the energy density, $\overline{P}$ the effective pressure , P the isotropic pressure, ρ_p stand for particle energy density, u^i the 4-velocity of the fluid, s is the heat flow vector satisfying $u^i u_i = -1, u^i s_i = u^i x_i = 0, x^i x_i = 1$. We assume that the heat flow is in x-direction only so that, s being a function of time. Also string to be lying along x- axis. In co-moving coordinate system the components of T_{ij} are

$$T^1_1 = \overline{P} - \lambda, T^2_2 = T^3_3 = \overline{P}, T^4_4 = -\rho, T^1_4 = s . \tag{6}$$

Now, let us explicitly write Barber's field equations for the line element (1) as follows

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}+\frac{h^2}{A^2}=-8\pi\phi^{-1}\overline{P}+8\pi\phi^{-1}\lambda\,, \tag{7}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{h^2}{A^2}=-8\pi\phi^{-1}\overline{P}\quad, \tag{8}$$

$$\frac{\ddot{B}}{B}+\frac{\ddot{A}}{A}+\frac{\dot{A}\dot{B}}{AB}-\frac{h^2}{A^2}=-8\pi\phi^{-1}\overline{P}\,, \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{h^2}{A^2}=8\pi\phi^{-1}\rho\,, \tag{10}$$

$$\frac{h^2}{A^2}\left(\frac{\dot{C}}{C}-\frac{\dot{B}}{B}\right)=-8\pi\phi^{-1}s\,, \tag{11}$$

where overhead dot indicate differentiation with respect to time. We have eight unknowns $\rho, \overline{P}, \lambda, \phi, s, A, B, C$ and five equations. So to get determinate solution we need one or more additional conditions connecting the unknown quantities. It should be note that there is no complete freedom to assume arbitrary relation to satisfy the above non linear equations appeared above. The relations would not give physically unrealistic results even though they are mathematically correct. We consider the relations which were assumed in the literature. They are as follows:

i) Relation between metric potentials A, B, C.

ii) Relation between scale factor $a(t)$ and scalar field $\phi(t)$. The power relation is discussed by Pavon et al.[37] and Maartens [38].

iii) Simple power function relation between bulk viscosity coefficient and energy density $\varsigma(t)=\varsigma_0\rho^{\beta}$ where ς_0, β are constant. We consider $\beta=1$ i.e. the radiating fluid. This radiating model is discussed by Murphy [39].

From equations (8) and (9), we have

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)=0\,. \tag{12}$$

Case I

We assume the relation between metric potential, which was assumed by Roy and Prasad [40] for solving differential equation of Bianchi type VI_0 metric.

$$A=\frac{B}{C}\quad. \tag{13}$$

Using equation (13) in equation (12), we get

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{B}^2}{B^2}+\frac{\dot{C}^2}{C^2}-2\frac{\dot{B}\dot{C}}{BC}=0\,. \tag{14}$$

Now we assume the following adhoc relation to make equation (14) solvable

$$\frac{\ddot{C}}{C}=0 \tag{15}$$

We should note here that the above adhoc relation gives us physically meaningful solution in the following form

$$C=mt+d\,, \tag{16}$$

where the parameters m ,d are constant of integration. We have linear behavior in time. Such solutions were previously obtained by Hajj Broutros[41], Ram and Singh[42] and Katore *et al.*[43]. Making use of equation (16) in equation (14), we get

$$\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{m^2}{(mt+d)^2}-\frac{2m}{mt+d}\frac{\dot{B}}{B}=0\,. \tag{17}$$

Let us consider $f=B^2$. Since the non linear differential equations are intricate. We can proceed to solve it by taking various resort techniques. Then, we obtain

$$\ddot{f}-\frac{2m}{mt+d}\dot{f}+\frac{2m^2}{(mt+d)^2}f=0\,. \tag{18}$$

For simplicity taking $T=mt+d$, equation (18) read as

$$T^2\frac{d^2f}{dT^2}-2T\frac{df}{dT}+2f=0\,, \tag{19}$$

Equation (19) gives us

$$f=B^2=c_1T+c_2T^2\,, \tag{20}$$

where c_1, c_2 are constant of integration. The metric potentials are obtained as

$$A=\frac{1}{T^2}\left(c_1T+c_2T^2\right), \tag{21}$$

$$C=T \tag{22}$$

The Bianchi type VI0 model takes the following form

$$ds^2=-dT^2+\frac{1}{T^4}(c_1T+c_2T^2)^2dx^2+(c_1T+c_2T^2)e^{-2hx}dy^2+T^2e^{2hx}dz^2 \tag{23}$$

The metric potentials *B, C* are increasing function of time whereas *A* is decreasing function of time. As $t\to 0,\ A,B,C\to const$ i.e. there is no initial singularity at big bang. As $t\to\infty, B,C\to\infty, A\to const.$. This is consistent with observations. It should be note here that as $t\to\infty$ the behavior of metric potentials *A,B,C* is different i.e. we do not have an isotropic regime at large time [44].

We use the relation between scale factor $a(t)$ and scalar field $\phi(t)$ [45] given by

$$\phi=ba^n\,. \tag{24}$$

The scalar field is obtained as

$$\phi = b\left(c_1 T + c_2 T^2\right)^{\frac{n}{3}}. \tag{25}$$

The scalar field is an increasing function of time for $b > 0$ and decreasing for $b < 0$. At $t = 0$, the scalar field becomes constant i.e. the scalar field has no initial singularity.

The effective pressure ($\overline{P}$), energy density (ρ), bulk viscous coefficient (ς), string tension density (λ), heat function (s), isotropic pressure (P), particle energy density (ρ_p) respectively are found to be

$$\overline{P} = \frac{m^2 c_1^2 b}{8\pi\left(c_1 T + c_2 T^2\right)^{\frac{6-n}{3}}} - \frac{m^2 c_1 b}{16\pi T\left(c_1 T + c_2 T^2\right)^{\frac{3-n}{3}}} + \frac{h^2 b T^2}{8\pi\left(c_1 T + c_2 T^2\right)^{\frac{3-n}{3}}}, \tag{26}$$

$$\rho = \frac{m^2 b\left(2c_2^2 T^2 - c_1^2\right) - 4h^2 b T^2\left(c_1 T + c_2 T^2\right)}{32\pi\left(c_1 T + c_2 T^2\right)^{\frac{6-n}{3}}}, \tag{27}$$

$$\varsigma = \varsigma_0\left[\frac{m^2 b\left(2c_2^2 T^2 - c_1^2\right) - 4h^2 b T^2\left(c_1 T + c_2 T^2\right)}{32\pi\left(c_1 T + c_2 T^2\right)^{\frac{6-n}{3}}}\right], \tag{28}$$

$$\lambda = \frac{b\left(m^2 c_2 + 2h^2 T^2\right)}{\left(c_1 T + c_2 T^2\right)^{\frac{6-n}{3}}}, \tag{29}$$

$$s = \frac{-b c_1 m h T^2}{16\pi\left(c_1 T + c_2 T^2\right)^{\frac{6-n}{3}}}, \tag{30}$$

$$P = \frac{m^2 c_1^2 b}{8\pi\left(c_1 T + c_2 T^2\right)^{\frac{6-n}{3}}} - \frac{m^2 c_1 b}{16\pi T\left(c_1 T + c_2 T^2\right)^{\frac{3-n}{3}}} + \frac{h^2 b T^2}{8\pi\left(c_1 T + c_2 T^2\right)^{\frac{3-n}{3}}} + \frac{\varsigma\left(c_1 + 2c_2 T\right)}{c_1 T + c_2 T^2}. \tag{32}$$

$$\rho_p = \frac{m^2 b\left(2c_2^2 T^2 - c_1^2 - 32\pi c_2\right) - 4h^2 b T^2\left(c_1 T + c_2 T^2 + 16\pi\right)}{32\pi\left(c_1 T + c_2 T^2\right)^{\frac{6-n}{3}}} \tag{33}$$

The energy conditions are $\rho, \rho_p \geq 0$ whereas the sign of string tension density is unrestricted. The string tension density may be positive, negative or zero [17]. Here, the energy conditions are satisfied. It is observed that energy density (ρ) and particle energy density (ρ_p) are decreasing functions of time. At *t* increases they gradually decrease and eventually tend to constant as $t \to \infty$. Therefore the universe may be steady stage in far future (figure 1, 3). From figure (2) it seems that the early universe was highly heated. It is cooling with increasing time. Singh [46] and Sahu[47] have obtained similar results. It should be note that when the string tension density is small the energy density and particle density are large and when it tends to zero they tend to constant. The bulk viscosity and heat play an important role in early stage of evolution of the universe. The string tension density is negative. Thus, the string phase of the universe disappears i.e. we have an anisotropic fluid of particle.

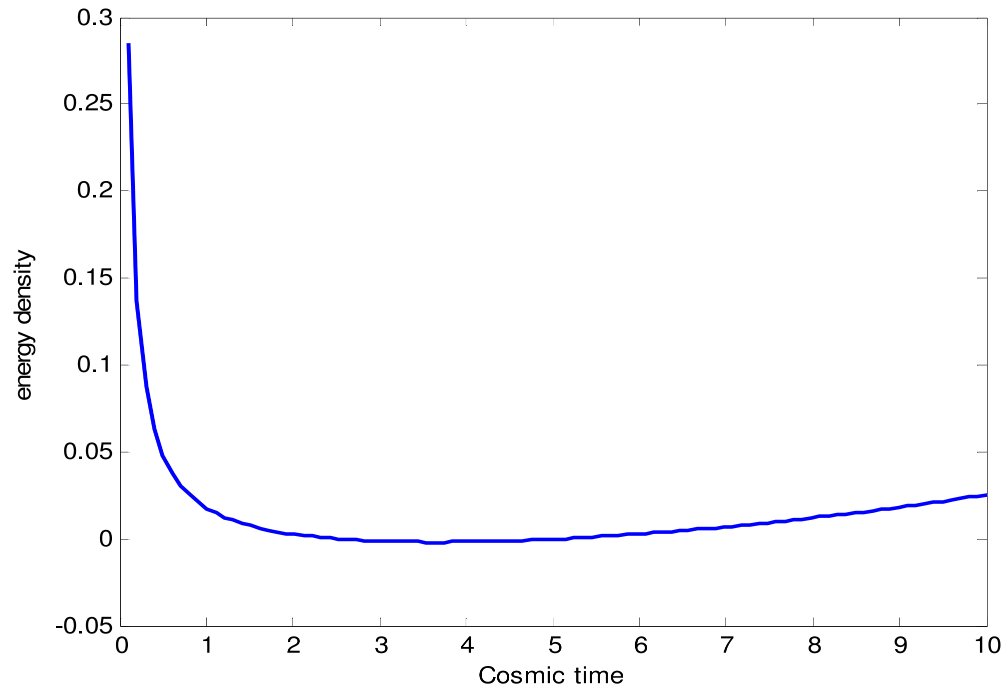

FIG. 1. Plot of energy density versus cosmic time with $c_1 = 3, c_2 = m = 1, h = 0.1, n = 3, b = -1$

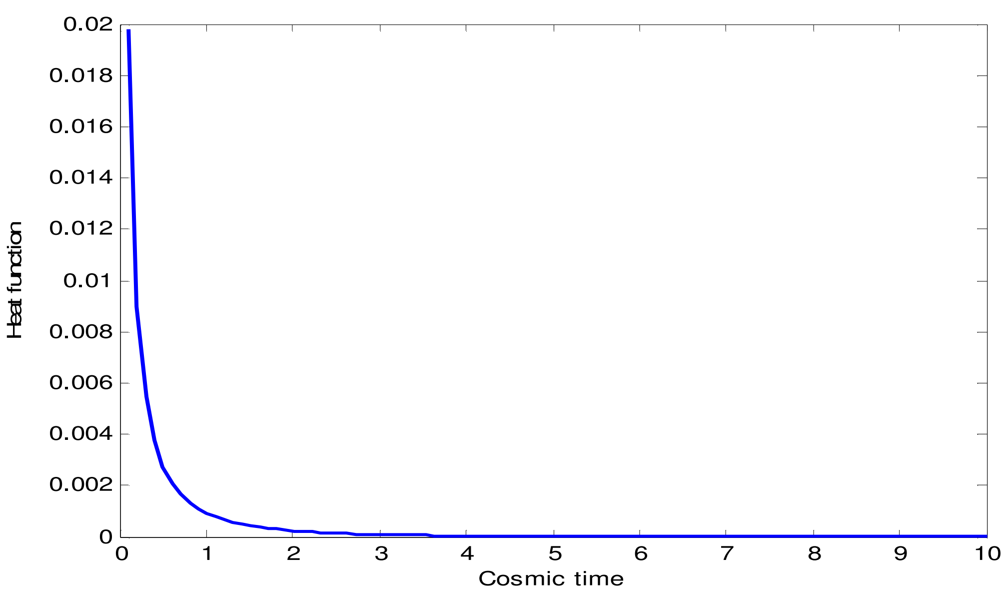

FIG.2. Plot of Heat versus cosmic time with $c_1 = 3, c_2 = m = 1, h = 0.1, n = -3, b = -1$

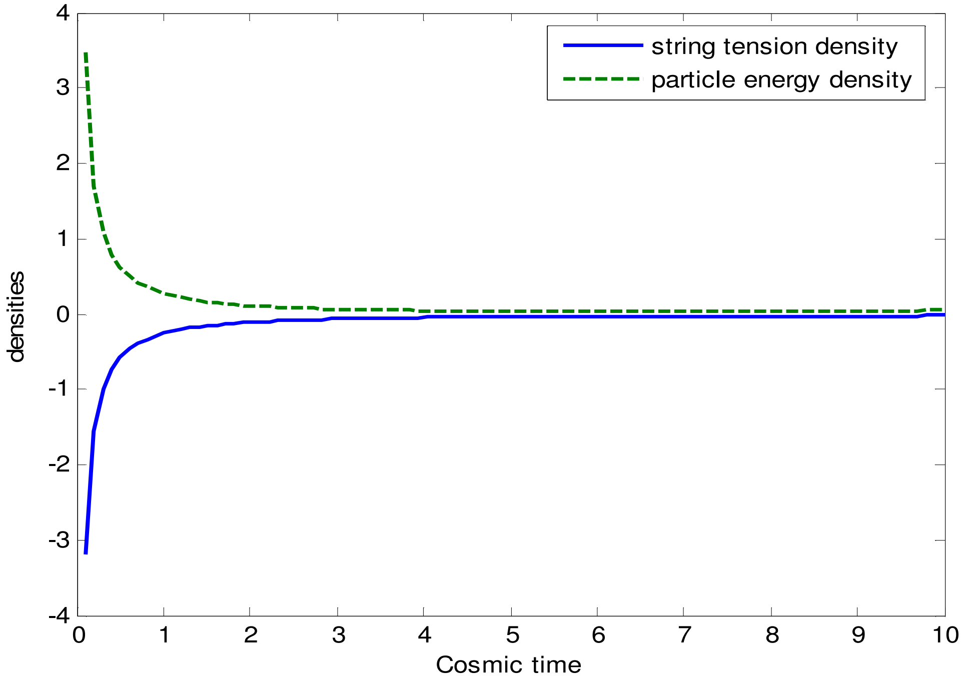

FIG.3. Plot of String tension density, particle energy density versus cosmic time with $c_1 = 3, c_2 = m = 1, h = 0.1, n = 3, b = -1$

The physical parameters such as volume (V), expansion scalar (θ), mean anisotropic parameter (Δ), shear scalar (σ) and deceleration parameter (q) respectively of the model becomes

$$V = ABC = c_1 T + c_2 T^2, \tag{34}$$

$$\theta = \frac{\dot{V}}{V} = \frac{c_1 + 2c_2 T}{c_1 T + c_2 T^2}, \tag{35}$$

$$\Delta = \frac{1}{3H^2}\sum_{i=1}^{3}(H_i - H)^2 = \frac{7c_1^2 + 10c_1c_2T + 4c_2^2T^2}{2(c_1 + 2c_2T)^2}, \tag{36}$$

$$\sigma^2 = \frac{7c_1^2 + 10c_1c_2T + 4c_2^2T^2}{12(c_1T + c_2T^2)^2} \quad , \tag{37}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{2(c_1^2 + c_1c_2T + c_2^2T^2)}{(c_1 + 2c_2T)^2}. \tag{38}$$

The volume of the universe is increasing with time. The positive sign of deceleration parameter means the universe is decelerating and negative sign stands for accelerating universe. The current observations of type Ia supernovae and cosmic microwave background radiation suggest that the universe is accelerating, but both do not altogether rule out the decelerating one which are also consistent with these observations [48]. Here, the deceleration parameter is positive throughout the evolution of the universe (figure 4). Thus, the universe decelerates in standard way which is consistent with observations. The expansion and shear scalar are decreasing functions of time (figure 5). The shear scalar tends to constant at large time. The shape of the universe was different in the initial time from the present. The rate of change of the shape of the universe gradually decreases. It is also evident that the universe is expanding with increase of time but the rate of expansion is slowing with passage of time. The ratio of shear scalar to expansion scalar is constant i.e. the anisotropy of the universe is maintained throughout the evolution of the universe. The results are similar to Pradhan *et al.* [19], Reddy and Naidu [8, 49].

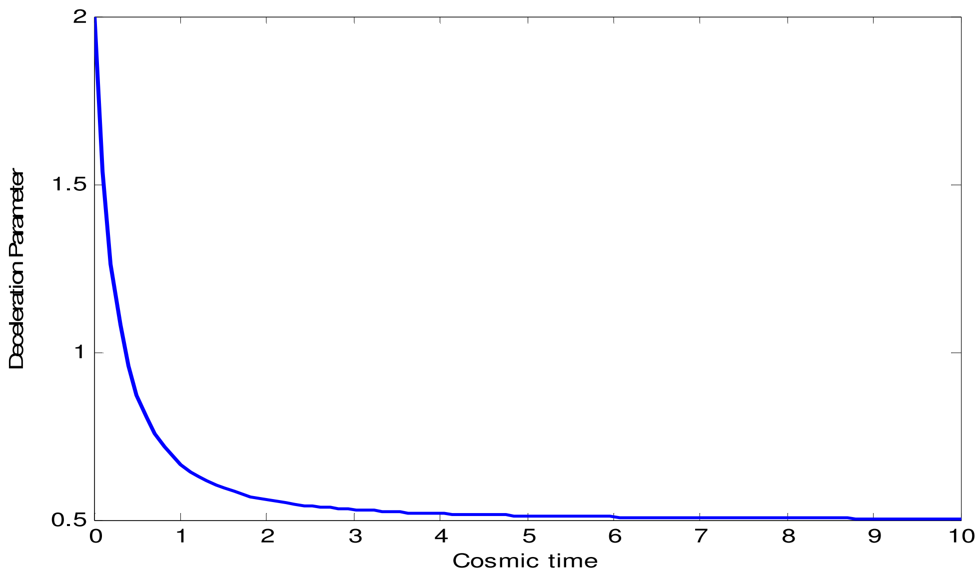

FIG.4. Plot of deceleration parameter versus cosmic time $c_1 = c_2 = 1$

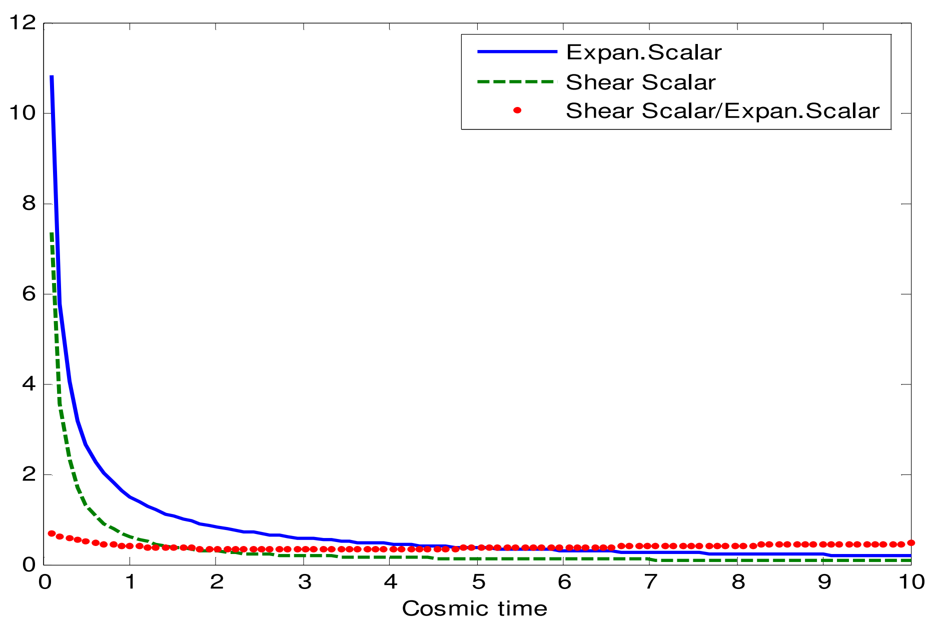

FIG.5. Plot of Expansion Scalar, Shear Scalar, Ratio of Shear Scalar to Expansion Scalar with $c_1 = c_2 = 1$

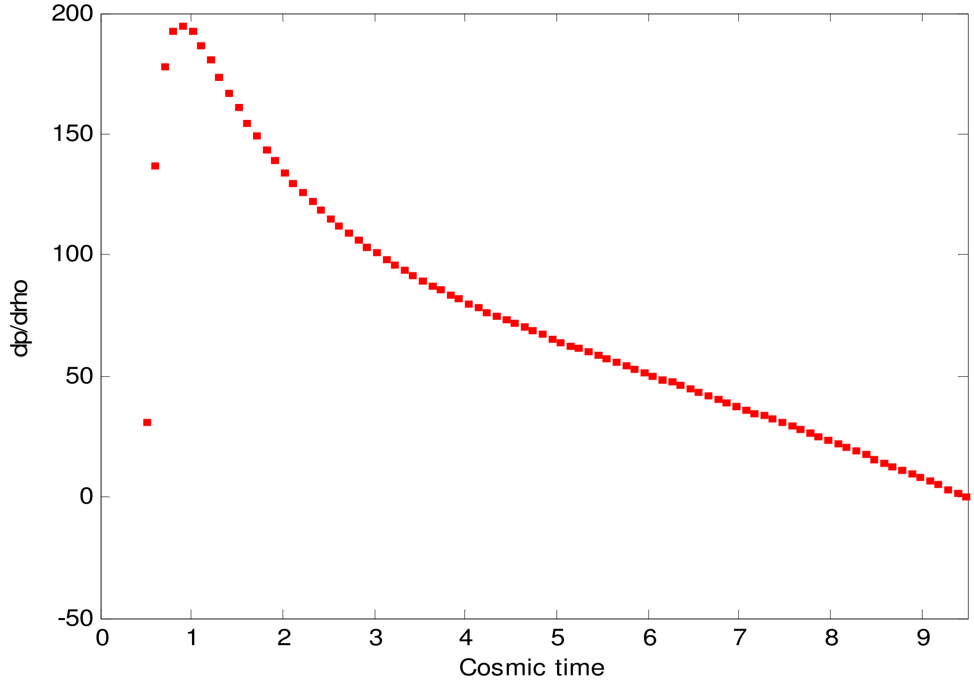

FIG.6. Plot of $dP/d\rho$ versus cosmic time $c_1 = 3, c_2 = m = 1, h = 0.1, n = 3, b = -1, \varsigma_0 = 200$.

We consider the stability by using the function $c^2 = dP/d\rho$. The stability of the model occurs when $c^2 \geq 0$. It is observed that $c^2 \geq 0$ throughout the evolution of the universe (figure 6). Therefore the model is stable. It should be note that in general relativity Sadeghi *et al*. [50] found that model is not stable at early epoch.

Case II

Now we consider another possible relation which is as follows

$$A = BC. \tag{39}$$

Using equation (39) and (12), we get

$$\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = \frac{\ddot{C}}{C} + \frac{\dot{C}^2}{C^2}. \tag{40}$$

Again we have a non linear equation so we assume adhoc relation in the following form

$$\frac{\ddot{C}}{C} + \frac{\dot{C}^2}{C^2} = 0. \tag{41}$$

Put $\dot{C} = F(C) \Rightarrow \ddot{C} = FF'$ then equation (41) reduces to

$$2FF' + \frac{2}{C}F^2 = 0. \tag{42}$$

Without loss of generality using equation (42) leads to

$$C = B = (2lt + l_2)^{\frac{1}{2}}, \tag{43}$$

$$A = 2lt + l_2 , \tag{44}$$

Where the parameters l, l_2 are constant of integration. At $t = 0$, the metric potentials A, B, C are constant. Thus, the model is free from singularity. As $t \to \infty, A, B, C \to \infty$. These are solutions of the field equations in the absence of heat.

The Bianchi type VI0 model takes the following form

$$ds^2 = -dt^2 + (2lt + l_2)^2 dx^2 + (2lt + l_2)(e^{-2hx} dy^2 + e^{2hx} dz^2) \tag{45}$$

The scalar field is obtained as

$$\phi = b(2lt + l_2)^{\frac{2n}{3}} . \tag{46}$$

The scalar field is increasing function of time. At $t = 0$, scalar field has no initial singularity. The result is similar to Reddy and Venkateswarlu [8].

The effective pressure ($\overline{P}$), energy density (ρ), string tension density (λ), bulk viscous coefficient (ς), isotropic pressure (P), particle energy density (ρ_p) respectively are obtained as

$$\overline{P} = \frac{b(h^2 - l^2)}{8\pi(2lt + l_2)^{\frac{6-2n}{3}}} , \tag{47}$$

$$\rho = \frac{b(-h^2 + 5l^2)}{8\pi(2lt + l_2)^{\frac{6-2n}{3}}} , \tag{48}$$

$$\lambda = \frac{b(h^2 - l^2)}{4\pi(2lt + l_2)^{\frac{6-2n}{3}}} , \tag{49}$$

$$\varsigma = \frac{\varsigma_0 b(-h^2 + 5l^2)}{8\pi(2lt + l_2)^{\frac{6-2n}{3}}} , \tag{50}$$

$$P = \frac{b(h^2 - l^2)}{8\pi(2lt + l_2)^{\frac{6-2n}{3}}} + \frac{bl\varsigma_0(5l^2 - h^2)}{2\pi(2lt + l_2)^{\frac{9-2n}{3}}} . \tag{51}$$

$$\rho_p = \frac{-3bh^2 + 7bl^2}{8\pi(2lt + l_2)^{\frac{6-2n}{3}}} \tag{52}$$

It is found that $\rho, \rho_p \geq 0$ i.e. the energy conditions are satisfied. The energy density and particle density are decreasing function of time (figure 7). As $t \to \infty, \rho, \rho_p \to const.$ i.e. the universe may be steady state in the future. Therefore the universe is dominated by matter throughout the evolution (figure 7). The string density is negative. Thus, we have an anisotropic fluid at all stage of evolution of the universe. The result is similar to Singh[17]. The term $c^2 = dP / d\rho > 0$ i.e. the model is stable (figure 8).

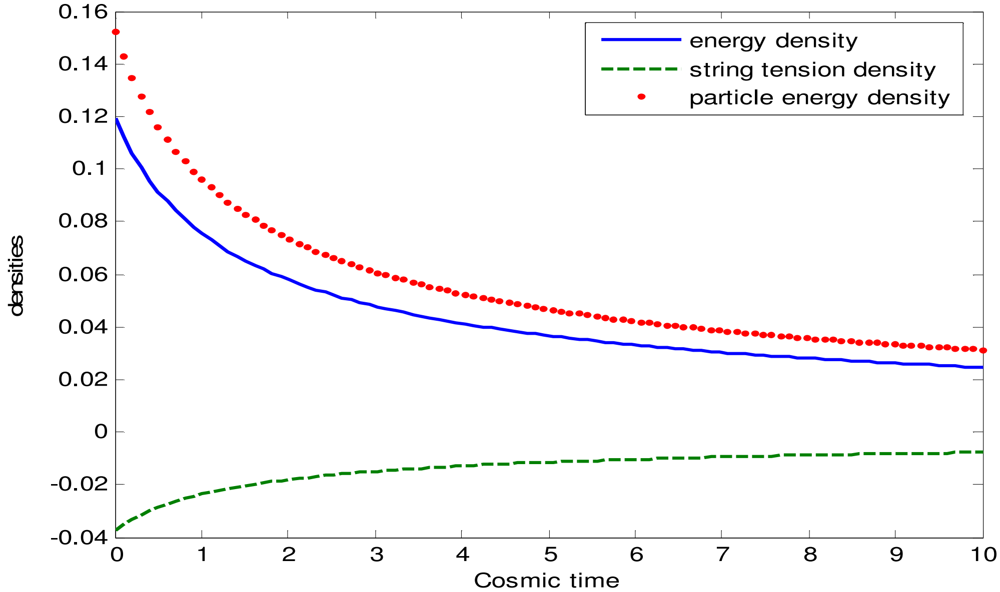

FIG.7. Plot of Energy density, String tension density, particle energy density versus cosmic time with $l=1, l_2=2, h=0.5, n=2$

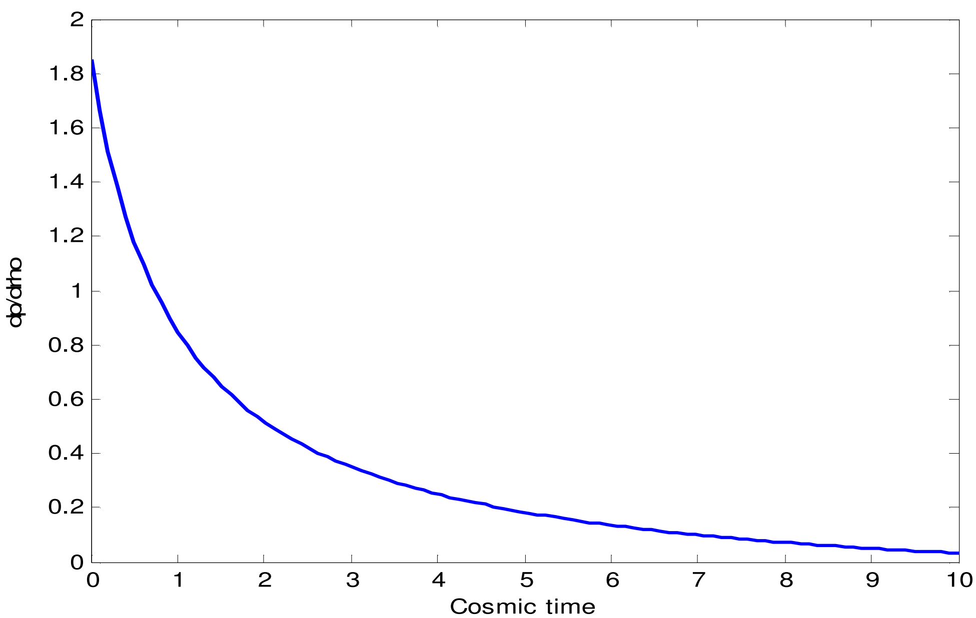

FIG. 8.Plot of $dP/d\rho$ versus cosmic time with $l=1, l_2=2, h=0.5, n=2, \varsigma_0=2$

The physical parameters such as volume (V), expansion scalar (θ), mean anisotropic parameter (Δ), shear scalar (σ) and deceleration parameter (q) respectively of the model found to be

$$V=(2lt+l_2)^2, \tag{53}$$

$$\theta=\frac{4l}{(2lt+l_2)}, \tag{54}$$

$$\Delta = \frac{1}{8}, \tag{55}$$

$$\sigma^2 = \frac{l^2}{3(2lt + l_2)^2}, \tag{56}$$

$$q = \frac{1}{2}. \tag{57}$$

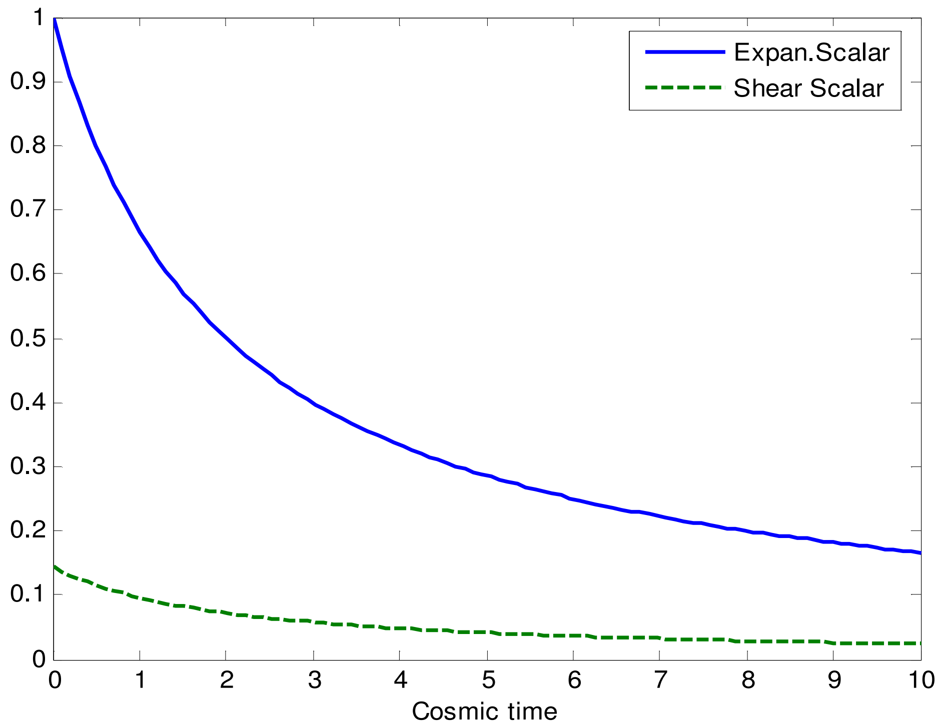

FIG. 9. Plot of Expansion Scalar, Shear scalar versus cosmic time with $l = 0.5, l_2 = 2$

The volume of the universe is increase indefinitely with time. The expansion scalar, shear scalar are decreasing function of time (figure 9). The universe is expanding with increase of time. The rate of expansion was large at early era and it decreases as time increases. The shear scalar was large at the time of big bang. Therefore, the shape of the universe was initially different than the present. Shear scalar tends to constant at large time. The universe is anisotropic. The deceleration parameter is positive i.e. the universe decelerates in standard way.

3. Conclusion

We have studied Bianchi type VI_0 in the context of Barbers second self creation theory of gravitation. We have considered bulk viscous string cosmological models together with heat flux. Two cases are analyzed by assuming different relation between metric potentials. It is found that both models are stable. The universe decelerates in standard way. The universe is anisotropic. The fluid is anisotropic. The shape of the universe was different at early era than the present. The rate of expansion of the universe slowing as time increases. The present cases admit anisotropy of the fluid which may be due to Barbers scalar field when compared with general relativity [51] and Brans-Dicke theory [52].

We emphasize here that in both investigated cases string phase disappears i.e. the strings are not important in the structure formation of the universe which is consistent with cosmic microwave background observation [53].

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