

# Omnidirectional reflection band in multi-layered graphite film based one dimensional photonic crystal nanostructure

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## Abstract

We report the omnidirectional reflection (ODR) in one dimensional photonic crystal (PC) structure consisting of alternate layers of Graphite as material of low refractive index and Tellurium (Te) as material of high refractive index. The effects of the incidence angles on the spectral reflectance have been investigated using transfer matrix method (TMM). The proposed structure gives 100% reflection within a wide range of wavelengths in the visible-near IR region and can be used effectively in wavelength filters, optical resonators and mirrors for visible-near IR region of electromagnetic spectrum. Due to considerable control of absorption of low frequency radiation in graphite, the damping and skin effect in the PC are also suppressed. The investigation has also been made for the study of role of ambient medium and the effect of number of layers in formation of ODR.

**Keywords:** Graphite photonics, photonic crystals, omnidirectional reflector, transfer matrix method, graphite based 1DPC, 1DPC

## 1. Introduction

Photonic Crystals (PCs) are optical nanostructures with a periodic modulation in the refractive index on the length scale comparable to optical wavelength. They are characterized by electromagnetic forbidden bands or photonic band gaps (PBGs). In other words, the propagation of electromagnetic waves, whose frequencies lie within the PBGs, is prohibited. This unique feature of the photonic crystal structures controls dramatically the flow of light within the structure and can lead to many potential applications in field of photonics [1-7]. The propagation of photons in PCs is analogous to propagation of electrons in semiconductor crystals where the effect of periodic refractive index in PCs is same as the effect of periodic potential function on propagation of electrons in semiconductors.

In these structures the refractive index is a periodic function in space and if the refractive index is periodic only in one dimension then the structure is called one dimensional photonic crystal (1DPC), if it is periodic in two dimensions and three dimensions then the structure is known as two dimensional photonic crystal (2DPC) and three dimensional photonic crystal (3DPC) respectively. PCs that work in microwave and far-infrared regions are relatively easier to fabricate. However, PCs that work in visible and the infrared (IR) regions, especially, 3DPC are difficult to fabricate because of their small lattice constants, which have to be comparable to the wavelength. Therefore, 1D PCs, which can easily be produced by the thin film deposition techniques, are preferable for use in the visible and IR regions. The simplest 1DPC is an alternating stack of two different mediums having reflection properties which find them used in variety of applications including high efficiency mirrors, Fabry Perot cavities, optical filters and feedback lasers. In recent time optical reflectors are one of the most widely used optical devices and a great deal of work has been done on the omnidirectional reflectors [8–14, 23, 24]. In metallic reflector, light can be reflected over a wide range of frequencies for arbitrary incident angles. However, at higher frequencies considerable amount of power is lost due to the absorption. In comparison to metallic reflectors, a graphite based multilayered nanostructures have high reflectivity in a certain range of frequencies, but the reflectivity is very sensitive to the incident angles. The range of reflected frequency of these reflectors can be enhanced by the appropriate selection of the material parameters and layer thickness.

In this paper we study the reflection properties of the multilayered 1DPC structure consisting alternate layers of Graphite and Tellurium. The theoretical analysis is based on the transfer matrix method [15]. Due to unusual properties of the band structure, electronic properties of graphite became the object of many recent experimental and theoretical studies [32-39]. Graphite is a gapless semiconductor with massless electrons and holes which have been described as Dirac-fermions [32-33, 40]. It was shown that in infrared and at larger wavelengths, transparency of graphite is defined by the fine structure constant [40].

The space-time dispersion of graphite conductivity was analyzed [41] and the optical properties of graphite were studied [42-44]. Thus, graphite has unique optical properties which make it useful in designing of opto-electronic devices. In the present study, we have neglected field absorption / attenuation as the constituent materials have negligible absorption coefficients in the wavelength range of interest. We observe that graphite based photonic crystal structure can be used as a good candidate for wavelength filter or broad reflector in the near infrared spectrum which is very useful in many imaging sensors in the field of optical technology. The paper is organized as follows. In Section 2 we develop the theoretical tool to analyze the ODR properties and linear characteristics of the 1D PC. In Section 3, we discuss the results obtained on the ODR properties of the graphite based 1DPC and the effect of ambient medium and number of layers. Finally, in Section 4, we conclude the paper.

## 2. Theoretical Analysis

To calculate the dispersion relation and reflection characteristics for the incident electromagnetic wave the Maxwell's equation is solved numerically by the transfer matrix method [15].

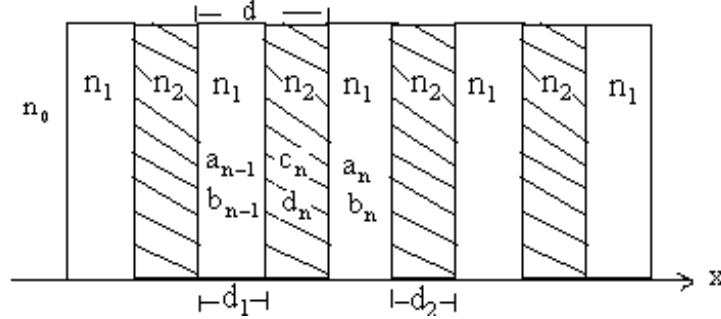


Figure 1. Periodic refractive index profile of the structure having refractive indices  $n_1$  and  $n_2$  respectively

The geometry of the structure under study is shown in the Fig. 1. Consider the propagation of EM wave along x-axis normal to the interface in one-dimensional system composed of periodic arrays of two different materials with a refractive index  $n_1$  and  $n_2$  and layer thickness  $d_1$  and  $d_2$ . The indices of refraction of the system are given as,

$$n(x) = \begin{cases} n_1, & 0 < x < d_1 \\ n_2, & d_1 < x < d_2 \end{cases} \quad (1)$$

with  $n(x) = n(x + d)$ . where  $d_1$  and  $d_2$  are the thicknesses of the layers and  $d = d_1 + d_2$  is the period of the structure. The electromagnetic field distribution within each layer can be expressed as the sum of right- and left-hand side propagating wave. The electric field within the both layers of the  $n$ th unit cell can be written as

$$E_1(x) = [(a_n e^{-ik_1(x-nd)} + (b_n e^{ik_1(x-nd)})] e^{i\omega t} \quad (2)$$

$$E_2(x) = [(c_n e^{-ik_2(x-nd)} + (d_n e^{ik_2(x-nd)})] e^{i\omega t} \quad (3)$$

Where,  $k_i = \left[ \left( \frac{n_i \omega}{c} \right)^2 - \beta^2 \right]^{\frac{1}{2}} = \frac{n_i \omega}{c} \cos \theta_i$ ,

$\theta_i$  is the ray angle in the  $i^{\text{th}}$  layer ( $i = 1, 2$ ),  $\beta$  is the propagation constant and  $n_i$  is the refractive index of the constituent layers. The coefficients  $a_n$ ,  $b_n$ ,  $c_n$  and  $d_n$  are related through the continuity boundary conditions at the interfaces  $x = (n - 1) d$  and  $x = (n - 1) d + d_2$ . This continuity condition leads to the matrix equations, which relates the coefficient in the first layer of the  $n^{\text{th}}$  cell, is given as

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = T_n \begin{pmatrix} a_n \\ b_n \end{pmatrix} \quad (4)$$

where  $T_n$  is called the transfer matrix given by

$$T_n = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad (5)$$

The matrix elements A, B, C and D are,

$$A = e^{ik_1 d_1} \left[ \cos k_2 d_2 + \frac{1}{2} i \left( \eta + \frac{1}{\eta} \right) \sin k_2 d_2 \right] \quad (6)$$

$$B = e^{-ik_1 d_1} \left[ \frac{1}{2} i \left( \eta - \frac{1}{\eta} \right) \sin k_2 d_2 \right] \quad (7)$$

$$C = e^{ik_1 d_1} \left[ -\frac{1}{2} i \left( \eta - \frac{1}{\eta} \right) \sin k_2 d_2 \right] \quad (8)$$

$$D = e^{-ik_1 d_1} \left[ \cos k_2 d_2 - \frac{1}{2} i \left( \eta + \frac{1}{\eta} \right) \sin k_2 d_2 \right] \quad (9)$$

The parameter  $\eta$  depends on the polarization. For the TE and TM polarizations,  $\eta$  is given by,

$$\eta_{\text{TE}} = \frac{k_1}{k_2} \quad (10)$$

and

$$\eta_{\text{TM}} = \frac{k_1 n_2^2}{k_2 n_1^2} \quad (11)$$

For finite stacks, the coefficient of right and left hand side propagating wave in both sides of the multilayer structure  $a_N$  and  $b_N$ , are calculated by multiplying transfer matrix of each cell as,

$$\begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = T_1 T_2 \dots T_N \begin{pmatrix} a_N \\ b_N \end{pmatrix} \quad (12)$$

where  $N$  is the total number of the cell. The coefficient of reflection is given by solving above matrix equation with the condition  $b_N = 0$  as,

$$r_N = \begin{pmatrix} b_0 \\ a_0 \end{pmatrix} \quad (13)$$

Thus the reflectivity (or reflectance) of the structure may be calculated as,

$$R_N = |r_N|^2 \quad (14)$$

Now, according to Bloch theorem, the electric field vector is of the form  $E(x) = E_{K(x)} e^{i(\omega t - Kx)}$  where  $E_{K(x)}$  is periodic with 'd'. For the determination of  $K$  as a function of eigenvalue, the equation is written as,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} a_n \\ b_n \end{bmatrix} = e^{iKd} \begin{bmatrix} a_n \\ b_n \end{bmatrix} \quad (15)$$

The solution of this matrix equation leads to the dispersion relation for the PC structure,

$$K(\omega) = \left( \frac{1}{d} \right) \cos^{-1} \left[ \cos(k_1 d_1) \cos(k_2 d_2) - \frac{1}{2} \left( \eta + \frac{1}{\eta} \right) \sin(k_1 d_1) \sin(k_2 d_2) \right] \quad (16)$$

The existence of omnidirectional photonic band gap in one dimensional photonic crystal requires the incident waves to be launched from vacuum or from a low refractive index ambient medium [5].

### 3. Result & Discussion:

We present the numerical analysis of Omni-directional reflection band in 1DPC with alternate layers of Graphite-Te. The value of refractive index of Graphite is taken as 2.87 and the refractive index of Te is taken as 4.6 respectively. The thickness of each layer is calculated by quarter wave stack condition  $d = \lambda_0 / 4n$ , where  $\lambda_0 = 800$  nm which is the central wavelength and  $n$  is the refractive index of layer. So the thickness of Graphite film is taken as 69.68 nm and that of Te is taken as 43.47 nm. The number of layers for computation is taken as  $N=20$ . The reflection band spectrum is shown for both TE and TM modes.

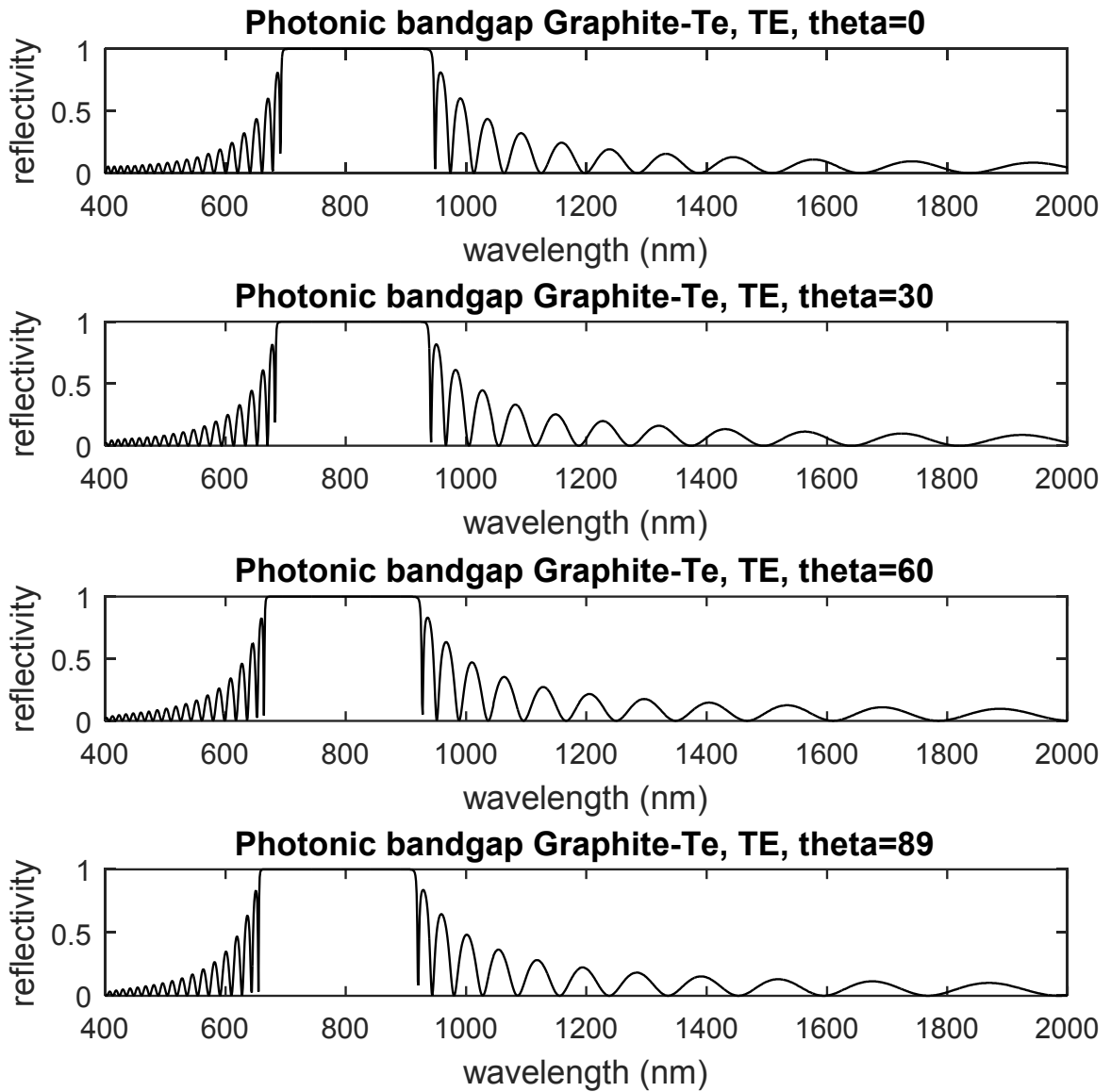


Fig 2(a). The reflectance spectra for [Graphite-Te] for TE modes showing the total reflection region and bandwidth for  $n_1=2.87$ ,  $n_2=4.6$ ,  $d_1=69.68$  nm,  $d_2=43.47$  nm and  $N=20$  at various incident angles  $\Theta=0^0$ ,  $\Theta=30^0$ ,  $\Theta=60^0$ ,  $\Theta=89^0$ .

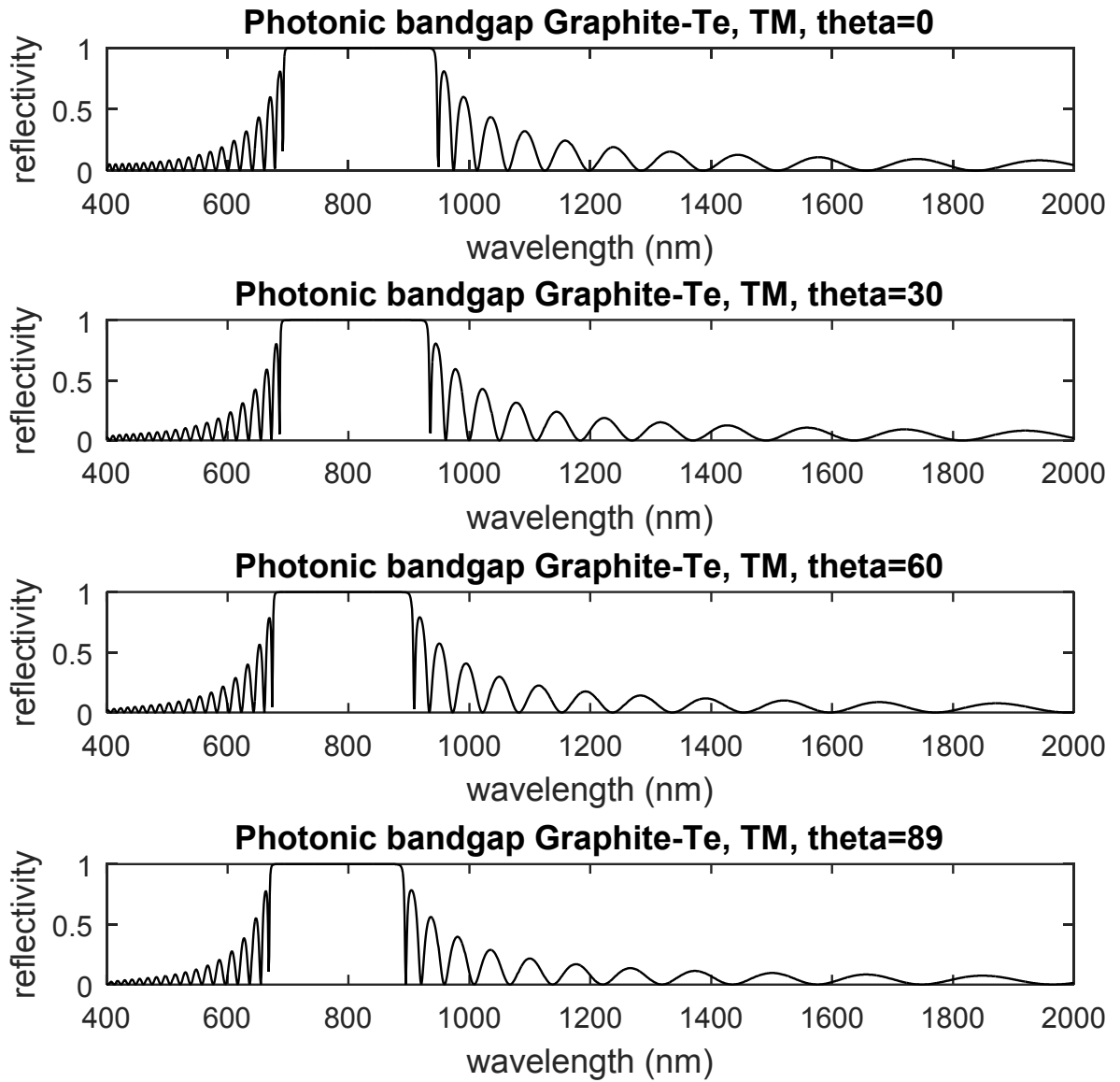


Figure 2(b). The reflectance spectra for [Graphite-Te] for TM modes showing the total reflection region and bandwidth for  $n_1=2.87$ ,  $n_2=4.6$ ,  $d_1=69.68$  nm,  $d_2=43.47$  nm and  $N=20$  at various incident angles  $\theta=0^\circ$ ,  $\theta=30^\circ$ ,  $\theta=60^\circ$ ,  $\theta=89^\circ$ .

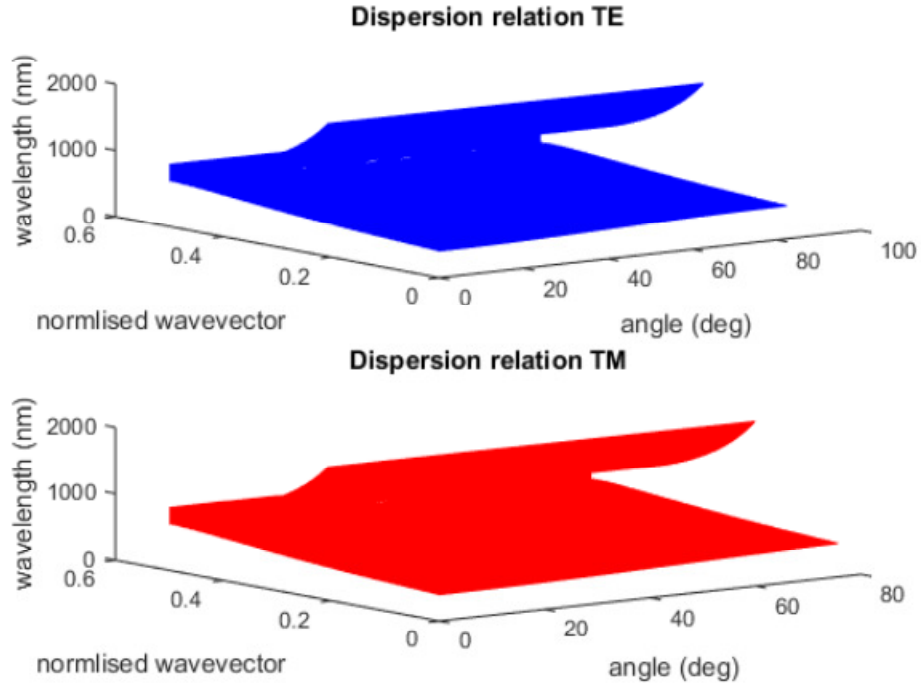


Figure 3: Dispersion relation with variation of angle of incidence (deg.) for TE modes (Blue lines) and TM modes (Red lines) for Graphite-Te.

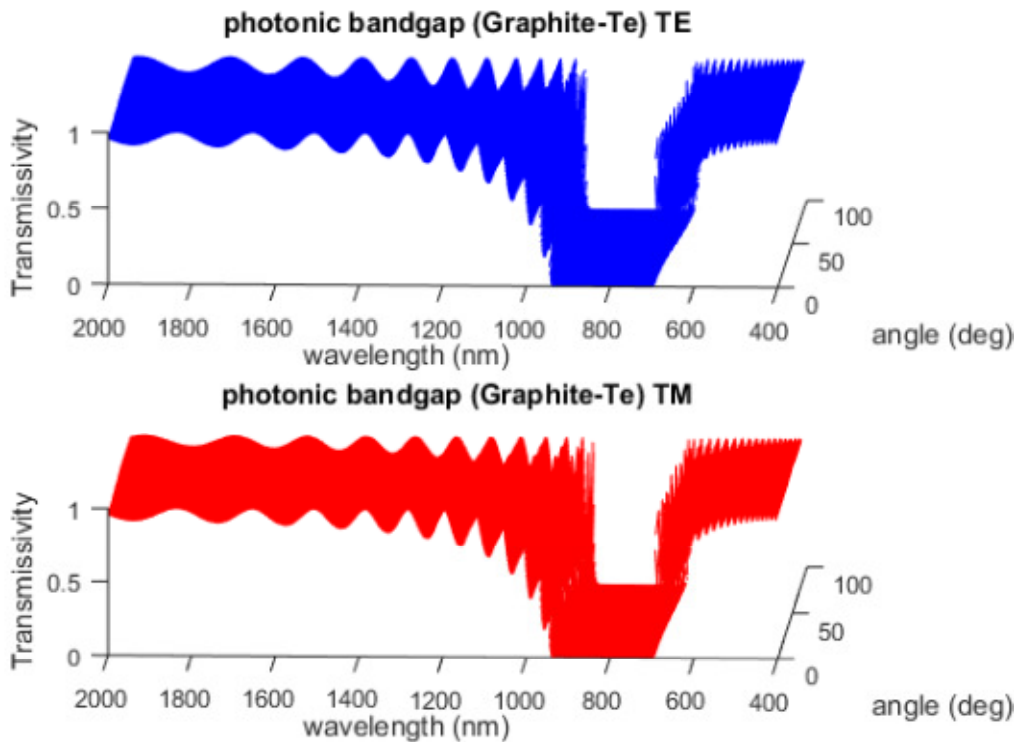


Figure 4: Transmissivity variation with wavelength (nm) and angle of incidence (deg.) for TE modes (Blue lines) and TM modes (Red lines) in multilayered Graphite-Te.

**Table 1: Total reflectance region band width for [Graphite-Te] at the various incident angles for TE and TM modes for N=20.**

[Graphite-Te] N=20	TE			TM		
	Lower Band Edge (nm)	Upper Band Edge (nm)	Bandgap (nm)	Lower Band Edge (nm)	Upper Band Edge (nm)	Band gap (nm)
$\Theta=0^\circ$	697	939	242	697	939	242
$\Theta=30^\circ$	688	932	244	691	926	235
$\Theta=60^\circ$	669	919	250	680	899	219
$\Theta=89^\circ$	660	912	252	674	885	211
Omni directional reflection band = 885 nm - 697 nm=188 nm						

It is observed from the above table as angle of incidence increases the bandgap for TE modes increases whereas the bandgap for TM modes decreases. There is a shift of bandgap towards smaller wavelength for TE and TM modes as we increase the angle of incidence. Hence the totally reflected wavelength band which is common to the both polarizations and for the entire incident angles lie from 885 nm to 697 nm which gives a significant omnidirectional reflection band of 188 nm. This narrow omnidirectional reflection band in near infrared region of electromagnetic spectrum makes it suitable for making near IR cutoff filters which can be used in digital cameras and other imaging sensors. Moreover we have studied the effect of ambient medium on this PC structure and analysis shows that the structure will give omnidirectional reflection band only if  $n_0 < n_1$ , where  $n_0$  is the refractive index of ambient medium and width of omnidirectional reflection band decreases and there is also shift of ODR towards lower wavelengths (blue shift) as  $n_0$  approaches towards  $n_1$ .



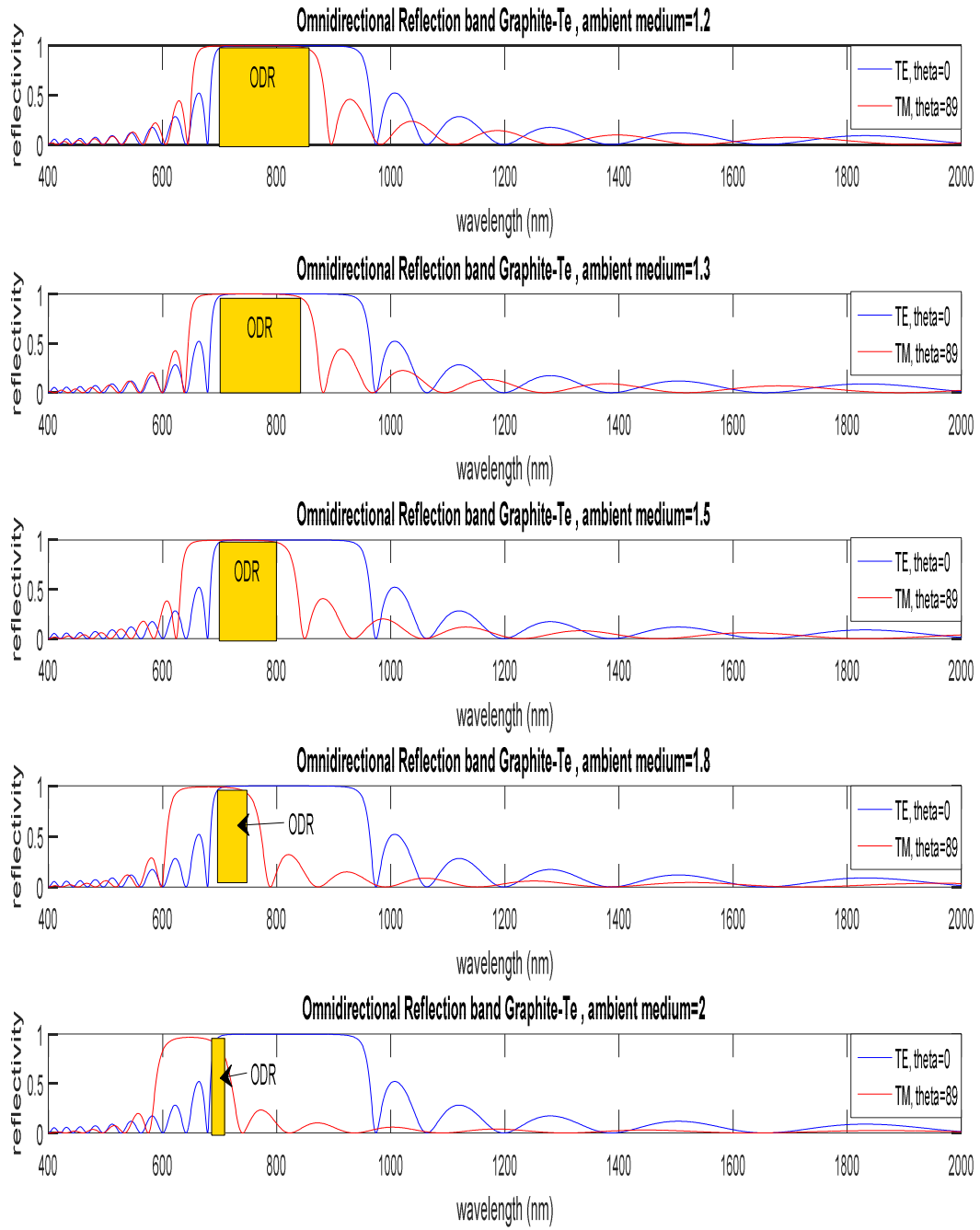


Figure 5. Effect of ambient medium in omnidirectional reflection band for [ Graphite-Te ] . Ambient medium refractive index  $n_0=1.2, 1.3, 1.5, 1.8, 2.0$  for  $N=10$ , for TE  $\theta=0^\circ$  (Blue lines) and TM  $\theta=89^\circ$  (Red lines)

The width of ODR band for [Graphite-Te] structure with  $N=10$  is shown with different ambient medium refractive index  $n_0=1.2, 1.3, 1.5, 1.8,$  and  $2.0$  are shown in Fig 5, but when  $n_0$  approaches to the value of lower refractive index  $2.87$ , the omnidirectional reflection band completely disappears. Hence it is concluded that the ambient medium plays an important role in formation of ODR. Finally we have studied the effect of number of layers and it is found that as number of layers increases the band becomes wider for the same refractive index contrast as shown in Fig 6. The effect of number of layers is shown with [Graphite-Te] structure with number of layers varying from  $N=2, 4, 6, 10$  and  $25$  in Fig 6 and it was found that as the number of layers increases, the width of ODR with 100% reflection regions increases.

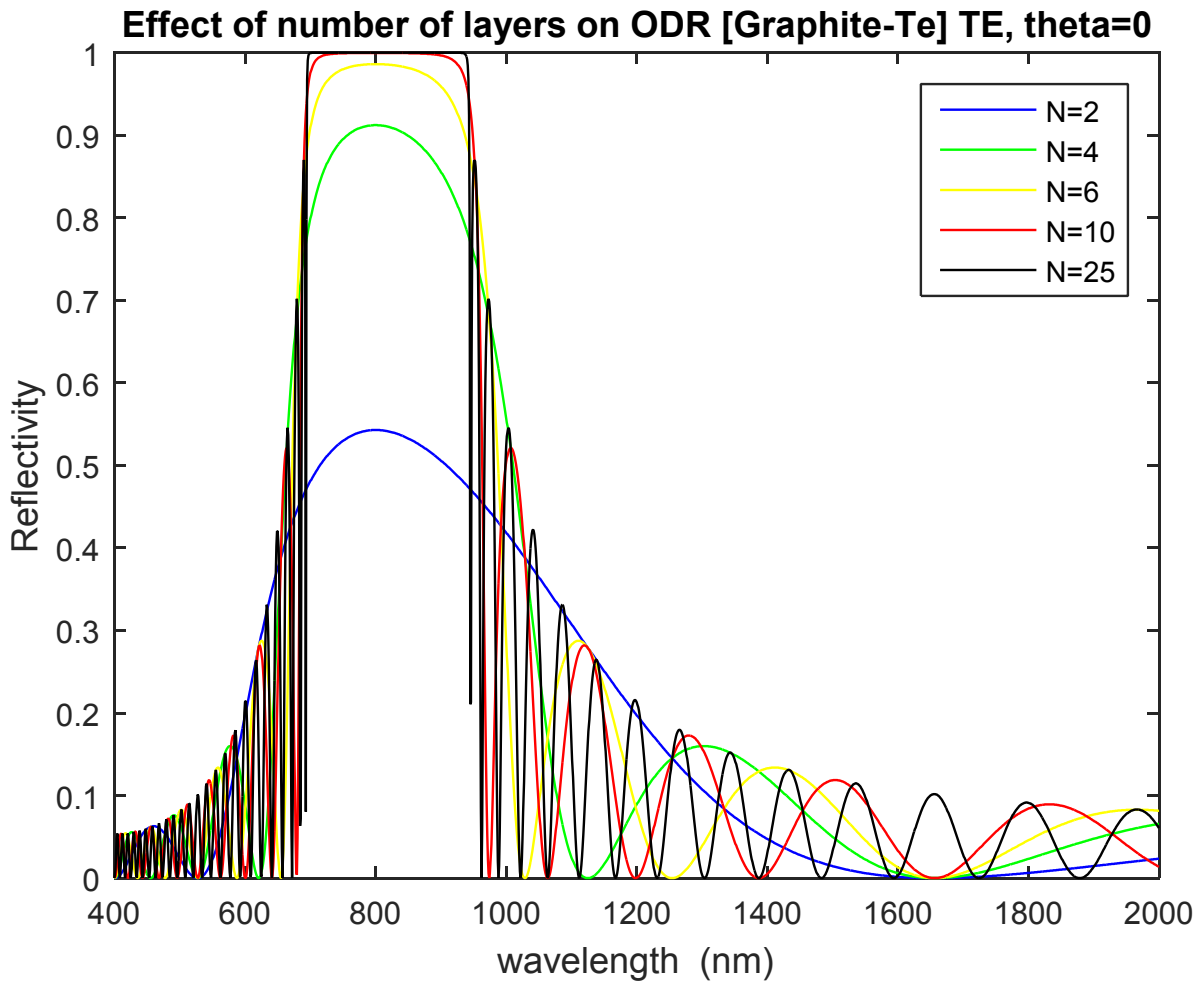


Figure 6. Effect of number of layers: The reflectance curve for [Graphite-Te] structure with  $N=2, 6, 10$  for TE modes for  $\Theta=0^\circ$ .

#### 4. Conclusion:

We have investigated theoretically the omnidirectional bandgap properties in proposed 20-period 1DPC using 69.68 nm thick graphite films and 43.47 nm thick Te films as alternate layers. The propagation properties and dispersion characteristics for both TE and TM modes for the structure of interest is analyzed using transfer matrix method and it is observed that there exists an omnidirectional band gap of 188 nm from (697 nm-885 nm) in the visible-near infrared region which makes this structure to be used as wavelength filters. The effect of ambient medium and the effect of number of layers are also analyzed and it is shown that the higher index ambient medium reduces the ODR but increasing the number of layers broadens the ODR. Hence they play major role in designing of ODR. We observe that Graphite based photonic crystal structure can be used as a good candidate for complete inhibition of transmission of frequency in near IR region. The proposed use of these multilayered nanostructures is as omnidirectional reflectors and near IR wavelength cutoff filters which are used in many imaging sensors in the field of optical technology.

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