Predicting the Fate of Schrodinger's Cat

James D. Stein

Department of Mathematics and Statistics
California State University, Long Beach, CA, USA

**Corresponding author E-mail*: james.stein@csulb.edu

(Received 20 January 2018, Accepted 16 February 2018, Published 23 February 2018)

Abstract

We describe a simple experiment in which a radioactive atom in a box decays in unit time with probability equal to ½, but such that the probability of a correct prediction of whether or not the atom decays is greater than ½.

***Keywords*:** Schrodinger's Cat, random variable, probability

1. Introduction

In the classic Schrodinger's Cat experiment, a radioactive atom which decays in unit time with probability equal to ½ is connected to a vial of poison gas. If the atom decays, the poison gas will be released, and the cat will die. This setup is shown in Fig. 1**.**

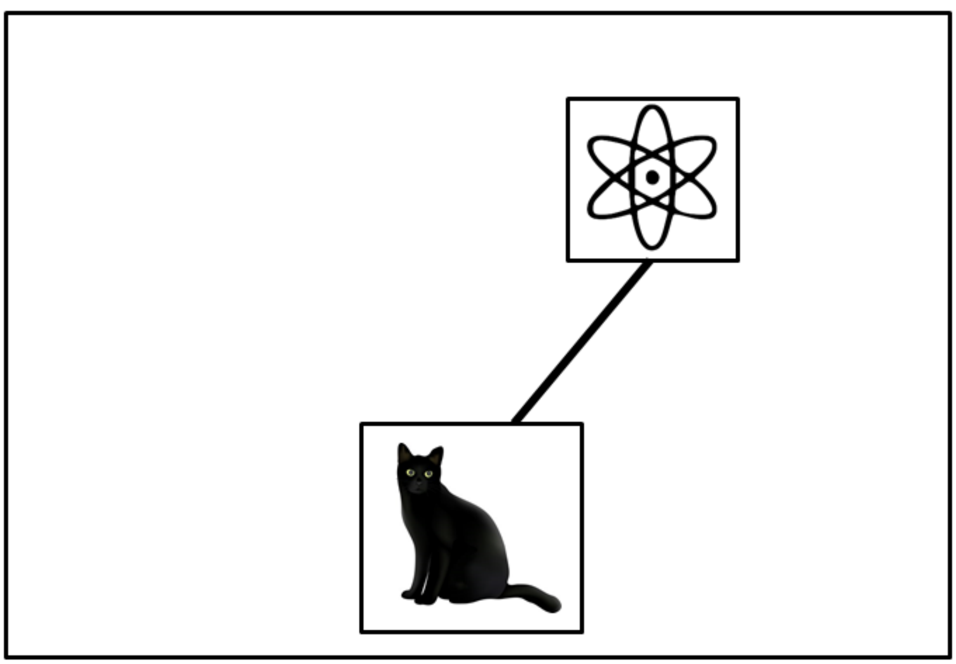

Fig. 1 – Schrodinger's Cat Experiment (Classic Version)

The great majority of interest in this experiment has centered on the state of the cat while the experiment is in progress. While this is certainly of interest to physicists, cat-lovers have been more concerned about whether the cat is dead or alive after the box is opened. It is generally accepted that strategies for predicting, before the box is opened, whether the cat will live or die do so correctly with probability equal to ½.

We show that a slight modification of the experimental apparatus produces the identical experiment from the standpoint of the experimenter, but a result of Blackwell [1] enables the experimenter to correctly predict the fate of the cat with probability greater than ½.

2. Description of the Modified Schrodinger's Cat Experiment

Fig. 2 shows the original setup of the experiment that is the focus of this paper. Two boxes are placed on a table of unit length. The left and right edges of the table can be viewed as being located at 0 and 1 on the real line. Box A contains a radioactive atom which decays with probability 2/3 in unit time, Box B contains a radioactive atom which decays with probability 1/3 in unit time. Boxes A and B are randomly positioned, but Box A, which contains the atom that decays with probability 2/3 in unit time, is always initially placed to the left of Box B. The solid dots in the two-headed line above the table in Fig. 2 below are the centers a and b of the two boxes, and are inserted in order to enable the reader to easily follow the simple computations. Boxes A and B are connected to the box containing the cat by pipes which allow the passage of poison gas.

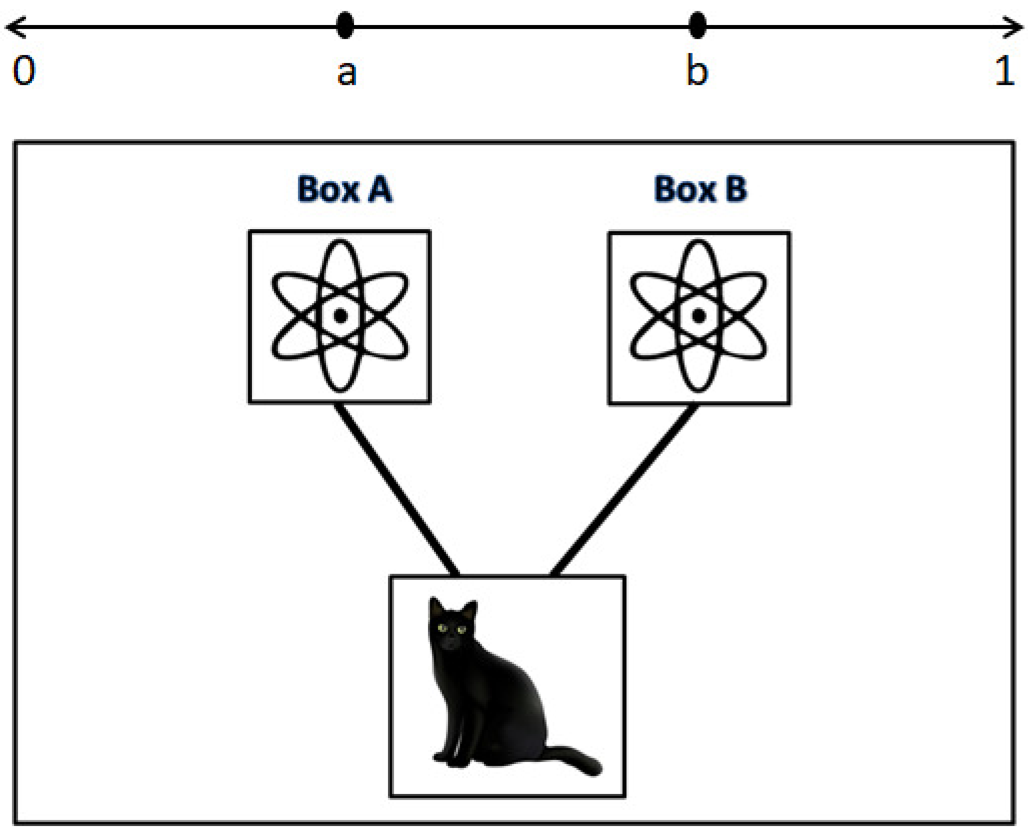

Fig. 2 – Initial Configuration for Modified Schrodinger's Cat Experiment

A fair coin is now flipped to remove one of the boxes. If the coin lands heads, Box A and connecting pipe are removed; if the coin lands tails, Box B and connecting pipe are removed. Fig. 3 shows the configuration after Box A has been removed.

The experimenter now enters the room, and sees only the apparatus as it appears in Fig. 3. There is no label to distinguish that this is Box B; that label is for the convenience of the reader. From the standpoint of the experimenter this appears identical to the classic Schrodinger's Cat experiment illustrated in Fig. 1. The experimenter now chooses a random number x between 0 and 1 (this could have been done prior to the experiment). If x, viewed as a distance from the left edge of the table, is to the left of the center of the remaining box (in this case, $x < b$), the experimenter predicts that the cat will live. If x is to the right of the center of the remaining box, the experimenter predicts that the cat will die.

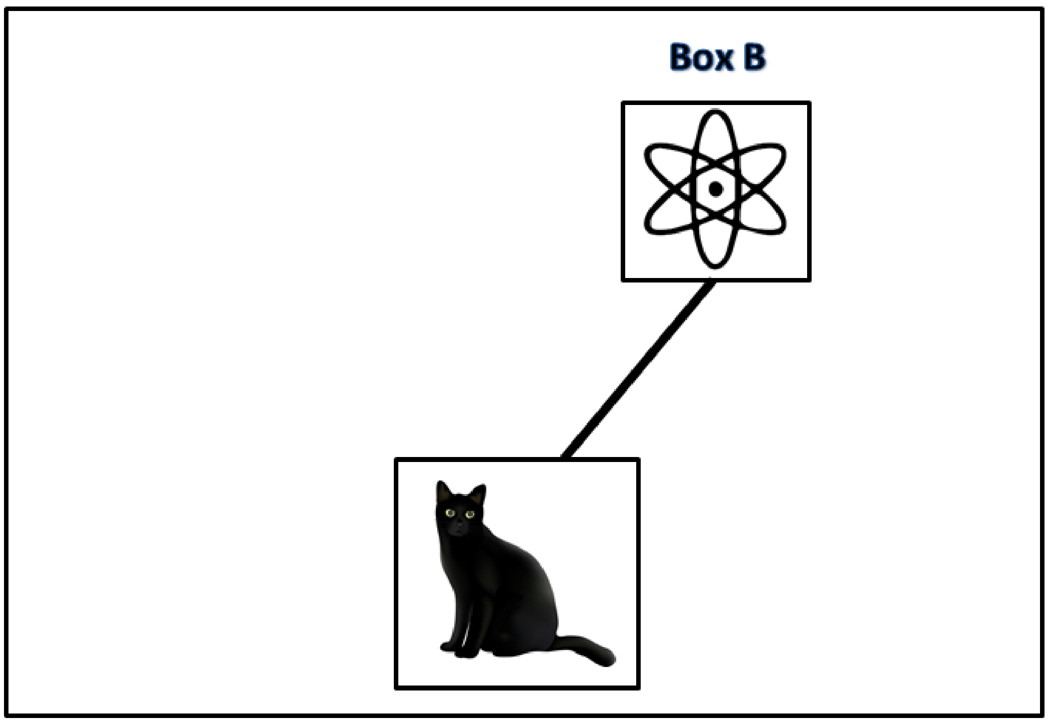

Fig. 3 – Modified Schrodinger's Cat Experiment after Box A is Removed

3. Probability Computations

Box A will remain on the table with probability ½, and the radioactive atom in it will decay (causing the cat to die) with probability 2/3. Since these events are independent, the probability that Box A will remain on the table and the atom in it will decay is therefore 1/2 x 2/3 = 1/3. Similarly, the probability that Box B will remain on the table and the atom in it will decay is 1/2 x 1/3 = 1/6. The probability that the cat will die is therefore 1/3 + 1/6 = ½.

The table below enables us to compute the probability of a correct prediction of the fate of the cat. Let p_a be the probability that $x < a$, and let p_b be the probability that $x > b$.

Note that $p_a + p_b < 1$.

Table 1 – Probabilities for Correct Prediction of Decay

Remaining Box	Atom Decays?	Variable Location Needed for Correct Prediction	Probability
A	Yes	x>a	½ x (2/3) x (1 – p_a)
A	No	x<a	½ x (1/3) x p_a
B	Yes	x>b	½ x (1/3) x p_b
B	No	x<b	½ x (2/3) x (1 – p_b)

The probability of a correct prediction of whether the atom decays or not is the sum of the four entries in the last column. By adding the numbers in the second and third columns to the numbers in the first and fourth columns, this sum is

$$\frac{1}{6}(p_a + p_b) + \frac{1}{3}(2 - (p_a + p_b)) = \frac{1}{3}\left(\frac{1}{2}(p_a + p_b)\right) + \frac{1}{3}(2 - (p_a + p_b)) \qquad [1]$$

$$= \frac{1}{3}\left(2 - \frac{1}{2}(p_a + p_b)\right) = \frac{2}{3} - \frac{1}{6}(p_a + p_b) > \frac{2}{3} - \frac{1}{6} = \frac{1}{2} \qquad [2]$$

So we can correctly predict the fate of the cat with probability greater than ½. We emphasize that the classic Schrodinger's Cat experiment in Fig. 1 and the one encountered in Fig. 3 are indistinguishable from the standpoint of the experimenter. In both, a box contains a radioactive atom which decays in unit time with probability equal to ½.

Although the random variable involved in this example was length as measured from the left edge of the table, there is an extensive list of physical parameters which would have served equally well. For instance, Boxes A and B could have been randomly colored, with Box A (the one containing the radioactive atom whose decay probability was 2/3) being the redder (in terms of frequency) of the two boxes. The random variable x could have been selected by picking a marble from a jar of colored marbles, and the atom would be predicted to decay if the color of the selected marble was bluer than the color of the remaining box.

4. Concluding Remarks

The reason that we can correctly predict the fate of the cat with probability greater than ½ is that the initial arrangement (using two boxes) is represented by a probability distribution. This experiment is an example of what is referred to as an extended Bernoulli Trial; some of the mathematics involved in extended Bernoulli Trials is investigated in [2].

There are two obvious directions for further investigation. The first is to see if there are other examples from physics which can be placed in this framework, and if anything can be derived from doing so. There may actually be one fairly well-known example; the solar neutrino deficit [3]. The author is insufficiently well-versed in physics to comment intelligently on this, but it appears to be related to the initial arrangement being a probability distribution, as in the modified Schrodinger's Cat experiment presented here.

The second is to use the idea of a random variable as a test to see whether or not there may be a "hidden variable" that is opaque to the experimenter. In the standard Schrodinger's Cat experiment, with a single box and radioactive atom which decays with probability ½ in unit time, it is shown in ([2]) that the probability of a correct prediction using a random variable and this technique is ½. If the use of a random variable produces a probability of a correct prediction in excess (or deficit) of what is expected, it suggests we might look for underlying probability distributions for the initial configuration.

Acknowledgements

The author would like to thank Professors Bruce Rothschild, Phil Everson, and Art Benjamin for enlightening conversations which led to the construction of this example. Special thanks are due to Professor Len Wapner, who originally brought Blackwell's result to the author's attention, and whose insight and ability to ask key questions eventually led to the result outlined in this paper.

REFERENCES

[1] David Blackwell, "On the Translation Parameter Problem for Discrete Variables," *Annals of Mathematical Statistics,* **22** (3), 393–399 (1951)

[2] James D. Stein, "Using Random Variables to Predict Experimental Outcomes," January 5th, 2018, arXiv:1801.02426

[3] https://en.wikipedia.org/wiki/Solar_neutrino_problem