Application of $\exp(-\varphi(\xi))$-expansion method for Tzitzeica type nonlinear evolution equations

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Abstract
The idea of $\exp(-\varphi(\xi))$-expansion method is used to construct new profuse exact traveling wave solutions of Tzitzeica type nonlinear evolution equations. By means of this method, three types of exact traveling wave solutions for each Tzitzeica type equations are obtained, including the hyperbolic functions and trigonometric functions. The obtained results show that $\exp(-\varphi(\xi))$-expansion method is very powerful, effective and convenient mathematical tool for non-linear evolution equations in science and engineering.

Keywords: $\exp(-\varphi(\xi))$-expansion method; Tzitzeica equation; Dodd-Bullough-Mikhailov equation; Tzitzeica-Dodd-Bullough equation; traveling wave solutions

1. Introduction
Largest part of the phenomena in real world can be demonstrated using non-linear evolution equations (NLEEs). Exact solutions of NLEEs plays an vital role, when we want to understand the physical mechanism of the phenomena such as the wave phenomena observed in fluid dynamics [7,8], plasma and elastic media [9,10] and optical fibers [11,12] etc. Recently, a number of prominent mathematicians and physicists have devoted considerable efforts in this interesting area of research. For instance the inverse scattering transform [1], the complex hyperbolic function method [2, 3], the rank analysis method [4], the Ansatz method [5, 6], the (G'/G)-expansion method [13-19], the modified simple equation method [20], the exp-functions method[21, 39], the sine-cosine method [22], the Jacobi elliptic function expansion method [23, 24], the F-expansion method [25, 26], the Backlund transformation method [27], the Darboux transformation method [28], the homogeneous balance method [29-31], the Adomian decomposition method [32, 33], the auxiliary equation method[34, 35], the $\exp(-\varphi(\eta))$-expansion method [36] and so on.

In this study, the objective is to add new traveling wave solutions of Tzitzeica equations in the literature. To do that we applied the $\exp(-\varphi(\xi))$-expansion method in Tzitzeica equations namely Dodd-Bullough-Mikhailov equation and Tzitzeica-Dodd-Bullough [37-38] which play a significant role in the scientific applications such as solid state physics, nonlinear optics , quantum field theory. The paper is arranged as follows. In section 2, we describe briefly the $\exp(-\varphi(\xi))$-expansion method. In section 3, we apply the method to the Tzitzeica type equations. Lastly in section 4, some conclusions are given.

2. The $\exp(-\varphi(\xi))$-expansion method

In the following, we will summarize the main steps of $\exp(-\varphi(\xi))$-expansion method. Consider a nonlinear equation, say in two independent variables x and t is given by

$$F(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, ...) = 0 \tag{1}$$

where u is an unknown function depending on x, t and P is a polynomial in $u(x,t)$ and its partial derivatives, in which the highest order derivatives and nonlinear terms are involved. The existing steps of method are as follows:

Step 1: Combining the independent variable x and t into one variable $\xi = kx + \omega t$, such that $u(x,t) = V(\xi)$, $\xi = kx + \omega t$ permits us reducing Eq.(1) to an ODE for $u = V(\xi)$

$$p(V, V', V'', ...) = 0 \tag{2}$$

Step 2: Suppose that the solution of ODE (2) can be expressed by a polynomial in $\exp(-\varphi(\xi))$as follows

$$V(\xi) = \sum_{i=0}^{m} \left(A_i Exp(-\varphi(\xi))\right)^i, \xi = kx + \omega t \tag{3}$$

where $\varphi'(\xi)$ satisfy the ODE in the form:

$$\varphi'(\xi) = \exp(-\varphi(\xi)) + \mu \exp(\varphi(\xi)) + \lambda \tag{4}$$

Then the solutions of ODE (4) are

When $\lambda^2 - 4\mu > 0, \mu \neq 0$,

$$\phi(\xi) = \ln\left(\frac{-\sqrt{(\lambda^2 - 4\mu)} \tanh\left(\frac{\sqrt{(\lambda^2 - 4\mu)}}{2}\right)(\xi + C) - \lambda}{2\mu}\right) \tag{5}$$

When $\lambda^2 - 4\mu > 0, \mu = 0$,

$$\phi(\xi) = -\ln\left(\frac{\lambda}{\exp(\lambda(\xi + E)) - 1}\right) \tag{6}$$

When $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0$,

$$\phi(\xi) = \ln\left(-\frac{2(\lambda(\xi + C) + 2)}{\lambda^2(\xi + C)}\right) \tag{7}$$

When $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0$,

$$\phi(\xi) = \ln(\lambda(\xi + C)) \tag{8}$$

When $\lambda^2 - 4\mu < 0$,

$$\phi(\xi)=\ln\left(\frac{\sqrt{(4\mu-\lambda^2)}\tan\left(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\xi+C)\right)-\lambda}{2\mu}\right) \tag{9}$$

$A_i,\omega,\lambda; i=0,1.......,m$ and μ are constants to be determined later, $A_m\neq 0$, the positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in ODE(3).

Step 3: By substituting Eq.(3) into Eq. (2) and using the ODE(4), collecting all terms with the same order $\exp(-\phi(\xi))$ together, the left hand side of Eq.(2) is convert into another polynomial in $\exp(-\phi(\xi))$ .Equating each coefficients of this polynomial to zero, yields a set of algebraic equations for $A_i,\omega,\lambda; i=0,1.......,m$ and μ.

Step 4: The constants $A_i,\omega,\lambda; i=0,1.......,m$ and μ can be found by solving the algebraic equations obtained in step 3. Since the general solutions of ODE (4) have been well known for us, then substituting $A_i,\omega; i=0,1.......,m$ with the general solutions of Eq. (4) into the Eq. (3), we have more traveling wave solutions of nonlinear evolution Eq. (1).

3. Application

In this section, we are going to demonstrate the $\exp(-\phi(\xi))$ -expansion method on three of well-known nonlinear evolution equations, namely, the Tzitzeica equation, Dodd-Bullough-Mikhailov (DBM) equation and Tzitzeica-Dodd-Bullough (TDB) equation. These nonlinear equations belong to a family of nonlinear equations which involve the exponential term e^{cu}, where $c\in\Re$.

3.1 Tzitzeica equation

Let us consider the Tzitzeica equation [37]:

$$u_{tt}-u_{xx}-e^{u}+e^{-2u}=0 \tag{10}$$

This equation plays a significant role in many scientific applications such as solid state physics, nonlinear optics and the quantum field theory. By the transformation $u=\ln(v)$;

Eq. (10) is changing into the following form:

$$vv_{tt}-v_t^2-vv_{xx}+v_x^2-v^3+1=0 \tag{11}$$

We would like to use the proposed method to obtain some new exact solutions of the Eq. (11), and therefore by using the transformation $u=\ln(v)$, the general soluton of Eq.(10) can be obtained easily. To obtain solution of Eq.(11), assume that

$$v=V(\xi),\ \xi=kx+\omega t \tag{12}$$

where k,ω are constants. Substitute Eq.(12) into Eq.(11) we get nonlinear ordinary differential equation

$$\left(\omega^2-k^2\right)VV''+\left(k^2-\omega^2\right)\left(V'\right)^2-V^3+1=0 \tag{13}$$

where prime denotes the differential with respect to ξ.

Balancing the highest order derivative with the nonlinear term of the highest order, we obtain $m=2$.

Therefore, the solution takes the following form:

$$V=A_0+A_1\left(Exp\left(-\varphi(\xi)\right)\right)+A_2\left(Exp\left(-\varphi(\xi)\right)\right)^2,A_2\neq 0 \tag{14}$$

where A_0, A_1 and A_2 are constants, to be determined later. Substituting Eq.(14) along with Eq.(4) into Eq.(13) and collecting all terms with the same order of $Exp(-\varphi(\xi))$, together the left-hand sides of Eq.(13) are converted into a polynomial in $Exp(-\varphi(\xi))$. Setting each coefficient of each polynomial to zero, we derive a set of algebraic equations for k, ω, A_0, A_1 and A_2, as follows:

$(Exp(-\varphi(\xi)))^6:\ 2\omega^2 A_2^2 - A_2^3 - 2k^2 A_2^2 = 0,$

$(Exp(-\varphi(\xi)))^5:\ -2k^2 A_2^2\lambda - 4k^2 A_1 A_2 + 4\omega^2 A_1 A_2 - 3A_1 A_2^2 + 2\omega^2 A_2^2\lambda = 0,$

$(Exp(-\varphi(\xi)))^4:\ -3A_1^2 A_2 + 6\omega^2 A_0 A_2 + 5\omega^2 A_1 A_2\lambda - k^2 A_1^2 - 5k^2 A_1 A_2\lambda - 3A_0 A_2^2 - 6k^2 A_0 A_2 + \omega^2 A_1^2 = 0,$

$(Exp(-\varphi(\xi)))^3:\ 2\omega^2 A_0 A_1 - k^2 A_1^2\lambda + 2\omega^2 A_1 A_2\mu + \omega^2 A_1 A_2\lambda^2 - k^2 A_1^2 + \omega^2 A_1^2 - 2\omega^2 A_2^2\lambda\mu - 3A_1^3$
$6A_0 A_1 A_2 - 2k^2 A_1 A_2\mu - 2k^2 A_0 A_1 + 2k^2 A_2^2\lambda\mu - k^2 A_1 A_2\lambda^2 - 10k^2 A_0 A_2\lambda + 10\omega^2 A_0 A_2\lambda = 0,$

$(Exp(-\varphi(\xi)))^2:\ -3k^2 A_0 A_1\lambda - 2\omega^2 A_2^2\mu^2 - 3A_0^2 A_2 + 3\omega^2 A_0 A_1\lambda - 4k^2 A_0 A_2\lambda^2 + k^2 A_1 A_2\lambda\mu + 4\omega^2 A_0 A_2\lambda^2$
$-\omega^2 A_1 A_2\lambda\mu - 3A_0 A_1^2 + 2k^2 A_2^2\mu^2 + 8\omega^2 A_0 A_2\mu - 8k^2 A_0 A_2\mu = 0,$

$(Exp(-\varphi(\xi)))^1:\ 2k^2 A_1 A_2\mu^2 - \omega^2 A_1^2\lambda\mu + 2\omega^2 A_0 A_1\mu - 3A_0^2 A_1 - 6k^2 A_0 A_2\lambda\mu - k^2 A_0 A_1\lambda^2$
$-2\omega^2 A_1 A_2\mu^2 + \omega^2 A_0 A_1\lambda^2 + k^2 A_1^2\mu\lambda + 6\omega^2 A_0 A_2\lambda\mu - 2k^2 A_0 A_1\mu = 0,$

$(Exp(-\varphi(\xi)))^0:\ 1 - \omega^2 A_1^2\mu^2 + k^2 A_1^2\mu^2 + 2\omega^2 A_0 A_2\mu^2 - A_0^3 - k^2 A_0 A_1\lambda\mu -$
$2k^2 A_0 A_1\mu^2 + \omega^2 A_0 A_1\lambda\mu = 0$

Solving this over-determined system with the assist of commercial computational software Maple, we have the following results.

Cluster-1: $A_0 = -\frac{2\mu+\lambda^2}{4\mu-\lambda^2},\ A_1 = -\frac{6\lambda}{4\mu-\lambda^2},\ A_2 = -\frac{6}{4\mu-\lambda^2}, \omega = \pm\sqrt{\left(\frac{4\mu k^2 - 3 - k^2\lambda^2}{4\mu-\lambda^2}\right)},$

Now substituting the values of A_0, A_1, A_2 and ω in the Eq.(14), using values of $\varphi(\xi)$ for each conditions and also using transformation $u(x,t) = \ln(V)$ the general solution of Eq.(10) can be obtained easily.

When $\lambda^2 - 4\mu > 0, \mu \neq 0$,

$$u(x,t) = \ln\left[\begin{array}{l} -\frac{2\mu+\lambda^2}{4\mu-\lambda^2} + \frac{6\lambda}{4\mu-\lambda^2} \times \left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+C)\right)+\lambda}\right) \\ -\frac{6}{4\mu-\lambda^2} \times \left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+C)\right)+\lambda}\right)^2 \end{array}\right] \tag{15}$$

When $\lambda^2 - 4\mu > 0, \mu = 0$,

$$u(x,t)=\ln\left[-\frac{2\mu+\lambda^2}{4\mu-\lambda^2}-\frac{6\lambda}{4\mu-\lambda^2}\times\left(\frac{\lambda}{\exp(\lambda(\xi+C))-1}\right)-\frac{6}{4\mu-\lambda^2}\times\left(\frac{\lambda}{\exp(\lambda(\xi+C))-1}\right)^2\right] \tag{16}$$

When $\lambda^2-4\mu<0$,

$$u(x,t)=\ln\left[\begin{array}{l}-\frac{2\mu+\lambda^2}{4\mu-\lambda^2}-\frac{6\lambda}{4\mu-\lambda^2}\times\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C)\right)+\lambda}\right)\\ -\frac{6}{4\mu-\lambda^2}\times\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C)\right)-\lambda}\right)^2\end{array}\right] \tag{17}$$

where $\xi=kx\pm\sqrt{\left(\frac{4\mu k^2-3-k^2\lambda^2}{4\mu-\lambda^2}\right)}.t$

3.2 Dodd-Bullough-Mikhailov equation

Now consider the Dodd-Bullough-Mikhailov equation given by [21-38]:

$$u_{xt}+e^{u}+e^{-2u}=0 \tag{18}$$

By the transformation $u=\ln(v)$; Eq. (18) is changing into the following form:

$$vv_{xt}-v_xv_t+v^3+1=0 \tag{19}$$

Therefore assume that

$$v=V(\xi),\xi=kx+\omega t \tag{20}$$

where k,ω are constants. Substitute Eq.(20) into Eq.(19) we get nonlinear ordinary differential equation

$$k\omega VV''-k\omega(V')^2+V^3+1=0 \tag{21}$$

where prime denotes the differential with respect to ξ.
Balancing the highest order derivative with the nonlinear term of the highest order, we obtain $m=2$.
Therefore, the solution of Eq.(21) takes the following form:

$$V=A_0+A_1(Exp(-\varphi(\xi)))+A_2(Exp(-\varphi(\xi)))^2, A_2\neq 0 \tag{22}$$

where A_0, A_1 and A_2 are constants, to be determined later. Substituting Eq.(22) along with Eq.(4) into Eq.(21) and collecting all terms with the same order of $Exp(-\varphi(\xi))$, together the left-hand sides of Eq.(21) are converted into a polynomial in $Exp(-\varphi(\xi))$.

Setting each coefficient of each polynomial to zero, we derive a set of algebraic equations for k, ω, A_0, A_1 and A_2, as follows:

$(Exp(-\varphi(\xi)))^6: \ A_2^3 + 2\omega k A_2^2 = 0,$

$(Exp(-\varphi(\xi)))^5: \ 4\omega k A_1 A_2 + 2\omega k^2 A_2^2 \lambda + 3A_1 A_2^2 = 0,$

$(Exp(-\varphi(\xi)))^4: \ 6k\omega A_0 A_2 + 3A_0 A_2^2 + k\omega A_1^2 + 3A_1^2 A_2 + 5k\omega A_1 A_2 \lambda = 0,$

$(Exp(-\varphi(\xi)))^3: \ -2k\omega A_2^2 \lambda\mu + 10k\omega A_0 A_2 \lambda + 2k\omega A_1 A_2 \mu + k\omega A_1^2 \lambda + 2k\omega A_1 A_2 \lambda^2$
$+ 2k\omega A_0 A_1 + 6A_0 A_1 A_2 + A_1^3 = 0,$

$(Exp(-\varphi(\xi)))^2: \ -k\omega A_1 A_2 \lambda\mu + 4k\omega A_0 A_2 \lambda^2 - 2k\omega A_2^2 \mu^2 + 3A_0 A_1^{\ 2} + 8k\omega A_0 A_2 \mu \quad + 3k\omega A_0 A_1 \lambda + 3A_0^2 A_2 = 0,$

$(Exp(-\varphi(\xi)))^1: \ 6k\omega A_0 A_2 \lambda\mu + k\omega A_0 A_1 \lambda^2 - 2k\omega A_1 A_2 \mu^2 - k\omega A_1^2 \lambda\mu + 2k\omega A_0 A_1 \mu + 3A_0^2 A_1 = 0$

$(Exp(-\varphi(\xi)))^0: \ 1 + A_0^3 + k\omega A_0 A_1 \lambda\mu - k\omega A_1^2 \mu^2 + 2k\omega A_0 A_2 \mu^2 = 0$

Solving this over-determined system with the assist of commercial computational software Maple, we have the following results.

Cluster-1: $A_0 = \dfrac{2\mu + \lambda^2}{4\mu - \lambda^2}, \ A_1 = \dfrac{6\lambda}{4\mu - \lambda^2}, \ A_2 = \dfrac{6}{4\mu - \lambda^2}, \omega = -\dfrac{3}{k(4\mu - \lambda^2)},$

Now substituting the values of A_0, A_1, A_2 and ω in the Eq.(22), using values of $\varphi(\xi)$ for each conditions and also using transformation $u(x,t) = \ln(V)$, we obtain the required solutions.

When $\lambda^2 - 4\mu > 0, \mu \neq 0$,

$$u(x,t) = \ln\left[\frac{2\mu + \lambda^2}{4\mu - \lambda^2} - \frac{6\lambda}{4\mu - \lambda^2} \times \left(\frac{2\mu}{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + C)\right) + \lambda}\right) + \frac{6}{4\mu - \lambda^2} \times \left(\frac{2\mu}{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + C)\right) + \lambda}\right)^2\right] \qquad (23)$$

where $\xi = kx - \dfrac{3}{k(4\mu - \lambda^2)} t$.

When $\lambda^2 - 4\mu > 0, \mu = 0$,

$$u(x,t) = \ln\left[-1 - \left(\frac{6}{\exp(\lambda(\xi + C)) - 1}\right) - 6\left(\frac{1}{\exp(\lambda(\xi + C)) - 1}\right)^2\right] \qquad (24)$$

where $\xi = kx + \dfrac{3}{k\lambda^2} t$.

When $\lambda^2 - 4\mu < 0$,

$$u(x,t) = \ln\left[-\frac{2\mu+\lambda^2}{4\mu-\lambda^2} - \frac{6\lambda}{4\mu-\lambda^2} \times \left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C)\right) - \lambda} \right) - \frac{6}{4\mu-\lambda^2} \times \left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C)\right) - \lambda} \right)^2 \right] \quad (25)$$

where $\xi = kx - \frac{3}{k(4\mu-\lambda^2)} t$.

Cluster-2: $A_0 = -\frac{(1\pm\sqrt{-3})(2\mu+\lambda^2)}{2(4\mu-\lambda^2)}$, $A_1 = -\frac{\lambda(1\pm\sqrt{-3})}{4\mu-\lambda^2}$, $A_2 = \frac{3(1\pm\sqrt{-3})}{4\mu-\lambda^2}$, $\omega = -\frac{3(1\pm\sqrt{-3})}{2k(4\mu-\lambda^2)}$,

Now substituting the values of A_0, A_1, A_2 and ω in the Eq.(22), using values of $\varphi(\xi)$ for each conditions and also using transformation $u(x,t) = \ln(V)$, we obtain the required solutions.

When $\lambda^2 - 4\mu > 0, \mu \neq 0$,

$$u(x,t) = \ln\left[-\frac{(1\pm\sqrt{-3})(2\mu+\lambda^2)}{2(4\mu-\lambda^2)} + \frac{\lambda(1\pm\sqrt{-3})}{4\mu-\lambda^2} \times \left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+C)\right) + \lambda} \right) - \frac{3(1\pm\sqrt{-3})}{4\mu-\lambda^2} \times \left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+C)\right) + \lambda} \right)^2 \right] \quad (26)$$

where $\xi = kx - \frac{3(1\pm\sqrt{-3})}{2k(4\mu-\lambda^2)} t$.

When $\lambda^2 - 4\mu > 0, \mu = 0$,

$$u(x,t) = \ln\left[\frac{1\pm\sqrt{-3}}{2} + \left(\frac{1\pm\sqrt{-3}}{\exp(\lambda(\xi+C))-1} \right) + 6\left(\frac{1}{\exp(\lambda(\xi+C))-1} \right)^2 \right] \quad (27)$$

where $\xi = kx + \frac{3(1\pm\sqrt{-3})}{2k\lambda^2} t$.

When $\lambda^2-4\mu<0$,

$$u(x,t)=\ln\left[\begin{array}{l}-\frac{(1\pm\sqrt{-3})(2\mu+\lambda^2)}{2(4\mu-\lambda^2)}-\frac{\lambda(1\pm\sqrt{-3})}{4\mu-\lambda^2}\times\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C)\right)-\lambda}\right)\\ -\frac{3(1\pm\sqrt{-3})}{4\mu-\lambda^2}\times\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C)\right)-\lambda}\right)^2\end{array}\right] \tag{28}$$

where $\xi=kx-\frac{3\left(1\pm\sqrt{-3}\right)}{2k\left(4\mu-\lambda^2\right)}t$.

3.3 Tzitzeica-Dodd-Bullough (TDB) equation

Finally, we would like to use the method to find out new solutions of Tzitzeica-Dodd-Bullough equation given by [39]:

$$u_{xt}-e^{-u}-e^{-2u}=0 \tag{29}$$

which plays a significant role in many scientific applications such as solid state physics, nonlinear optics and the quantum field theory. By the transformation $v(x,t)=e^{-u}$; Eq. (29) is changing into the following form:

$$-vv_{xt}+v_xv_t-v^3-v^4=0 \tag{30}$$

Therefore, introducing a complex variable ξ defined as

$$v=V(\xi),\ \xi=kx+\omega t \tag{31}$$

where k,ω are constants. Substitute Eq.(31) into Eq.(30) we get nonlinear ordinary differential equation

$$-k\omega VV''+k\omega(V')^2-V^3-V^4=0 \tag{32}$$

where prime denotes the differential with respect to ξ.

Balancing the highest order derivative with the nonlinear term of the highest order, we obtain $m=1$. Therefore, the solution of the Eq.(32) takes the following form:

$$V=A_0+A_1\left(Exp\left(-\varphi(\xi)\right)\right),\ A_1\neq 0 \tag{33}$$

where A_0 and A_1 are constants, to be determined later. Substituting Eq.(33) along with Eq.(4) into Eq.(32) and collecting all terms with the same order of $Exp\left(-\varphi(\xi)\right)$, together the left-hand sides of Eq.(32) are converted into a polynomial in $Exp\left(-\varphi(\xi)\right)$. Setting each coefficient of each polynomial to zero, we derive a set of algebraic equations for k,ω,A_0 and A_1, as follows:

$\left(Exp\left(-\varphi(\xi)\right)\right)^4$: $-k\omega A_1^2-A_1^4=0,$

$\left(Exp\left(-\varphi(\xi)\right)\right)^3$: $-4A_0A_1^3-2k\omega A_0A_1-A_1^3-k\omega A_1^2\lambda=0,$

$\left(Exp(-\varphi(\xi))\right)^2: \ -3A_0A_1^2-6A_0^2A_1^2-3k\omega A_0A_1\lambda=0,$

$\left(Exp(-\varphi(\xi))\right)^1: \ -2k\omega A_0A_1\mu-3A_0^2A_1-k\omega A_0A_1\lambda^2-4A_0^3A_1+k\omega A_1^2\lambda\mu=0$

$\left(Exp(-\varphi(\xi))\right)^0: \ -A_0^3-k\omega A_0A_1\lambda\mu+k\omega A_1^2\mu^2-A_0^4=0$

Solving this over-determined system with the assist of software Maple, we have the following results.

Cluster-1: $A_0=\pm\frac{1}{2}\sqrt{\frac{1}{\lambda^2-4\mu}}-\frac{1}{2},\ A_1=\pm\sqrt{\frac{1}{\lambda^2-4\mu}},\ \omega=\frac{1}{k\left(4\mu-\lambda^2\right)}.$

Now substituting the values of A_0, A_1 and ω in the Eq.(33), using values of $\varphi(\xi)$ for each conditions and also using transformation $u(x,t)=-\ln(V)$ the general solution of Eq.(29) can be obtained easily.

When $\lambda^2-4\mu>0,\mu\neq 0$,

$$u(x,t)=-\ln\left[\pm\frac{1}{2}\sqrt{\frac{1}{\lambda^2-4\mu}}-\frac{1}{2}\pm\sqrt{\frac{1}{\lambda^2-4\mu}}\times\left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+C)\right)+\lambda}\right)\right] \tag{34}$$

where $\xi=kx+\frac{1}{k\left(4\mu-\lambda^2\right)}t.$

When $\lambda^2-4\mu>0,\mu=0$,

$$u(x,t)=-\ln\left[\pm\frac{1}{2\lambda}-\frac{1}{2}\pm\frac{1}{\exp(\lambda(\xi+C))-1}\right] \tag{35}$$

where $\xi=kx-\frac{1}{k\lambda^2}t.$

When $\lambda^2-4\mu<0$,

$$u(x,t)=-\ln\left[\pm\frac{1}{2}\sqrt{\frac{1}{\lambda^2-4\mu}}-\frac{1}{2}\pm\sqrt{\frac{1}{\lambda^2-4\mu}}\times\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C)\right)-\lambda}\right)\right] \tag{36}$$

where $\xi=kx+\frac{1}{k\left(4\mu-\lambda^2\right)}t.$

4. Conclusion

In this article, the $\exp(-\varphi(\xi))$-expansion method has been successfully implemented to find new traveling wave solutions of Tzitzeica type equations. As a result, we obtained plentiful new exact solutions including trigonometric function, hyperbolic function and rational solutions. We hope that they will be useful for further studies in applied sciences. It is shown that the performance of this method is productive, effective and well-built mathematical tool for solving nonlinear evolution equations.

REFERENCES

[1] M.J. Ablowitz and P. A. Clarkson, "Soliton, nonlinear evolution equations and inverse scattering", (Cambridge University Press, New York), (1991)

[2] E.M.E. Zayed, A. M. Abourabia, K. A. Gepreel, M. M. Horbaty, "On the rational solitary wave solutions for the nonlinear HirotaCSatsuma coupled KdV system", *Appl. Anal.*, **85**, 751-768, (2006)

[3] K.W. Chow, "A class of exact periodic solutions of nonlinear envelope equation", *J. Math. Phys.*, **36**, 4125-4137, (1995)

[4] X. Feng, "Exploratory approach to explicit solution of nonlinear evolutions equations", *Int. J. Theo. Phys.*, **39**, 207-222, (2000)

[5] J. L. Hu, "Explicit solutions to three nonlinear physical models", *Phys. Lett. A*, **287**, 81-89, (2001)

[6] J. L. Hu, "A new method for finding exact traveling wave solutions to non-linear partial differential equations", *Phys. Lett. A*, **286**, 175-179, (2001)

[7] I.E. Inan and D. Kaya, "Exact Solutions of Some Nonlinear Partial Differential Equations", *Physica A*, **381(1),** 104-115, (2007) doi:10.1016/j.physa.2007.04.011

[8] A.K. Ray and J. K. Bhattacharjee, "Standing and Travelling Waves in the Shallow-Water Circular Hydraulic Jump", *Phys. Lett. A*, **371** (3), 241-248, (2007)

[9] P.M. Jordan and A. A. Puri, "Note on Traveling Wave Solutions for a Class of Nonlinear Viscoelastic Media", *Phys. Lett., A*, **335(2-3),** 150-156, (2005) doi:10.1016/j.physleta.2004.11.058.

[10] V.A. Osipov, "An Exact Solution for a Fractional Disclination Vortex", *Physics Letters A,* **193(1),** 97-101, (1994) doi:10.1016/0375-9601(94)00664-4

[11] K. Nakkeeran, "Optical Solitons in Erbium Doped Fibers with Higher Order Effects", *Physics Letters A,* **275(5-6**), 415-418, (2000) doi:10.1016/S0375-9601(00)00600-9.

[12] Z.Y. Yan, "Generalized Method and Its Application in the Higher-Order Nonlinear Schrodinger Equation in Nonlinear Optical Fibres", *Chaos Solitons and Fractals*, **16(5),** 759-766, (2003) doi:10.1016/S0960-0779(02)00435-6.

[13] M.L. Wang, X.Z. Li, J. Zhang, "The (G'/G)-expansion method and traveling wave solutions of nonlinear evolution equations in mathematical physics", *Phys. Lett. A,* **372,** 417-423, (2008)

[14] M.N. Alam, M.A. Akbar and S.T. Mohyud-Din, "A novel (G'/G)-expansion method and its application to the Boussinesq equation", *Chin. Phys. B,* Article ID 131006, 23(2) 02XXXX, (2014)

[15] H.O. Roshid, M.N. Alam, M.F. Hoque and M.A. Akbar, "A new extended (G'/G)-expansion method to find exact traveling wave solutions of nonlinear evolution equations", *Mathematics and Statistics,* **1(3),** 162-166, (2013)

[16] M.N. Alam and M.A. Akbar, "Exact traveling wave solutions of the KP-BBM equation by using the new generalized (G'/G)-expansion method", Springer Plus, 2(1), 617, (2013) DOI: 10.1186/2193-1801-2-617

[17] M.N. Alam, M.A. Akbar and H.O. Roshid, "Study of nonlinear evolution equations to construct traveling wave solutions via the new approach of generalized (G'/G)-expansion method", *Mathematics and Statistics*, **1(3),** 102-112,(2013) DOI: 10.13189/ms.2013.010302.

[18] M.A. Akbar, N. H. M. Ali and E.M.E. Zayed, "A generalized and improved (G'/G)-expansion method for nonlinear evolution equations", Math. Prob. Engr., 2012 (2012) doi: 10.1155/2012/459879.

[19] H. O. Roshid, M. R. Nizhum and M. A. Akbar, "Traveling waves solutions of nonlinear Klein-Gordon equation by extended (G'/G)-expansion method", *Annals of Pure and Appl. Math.*, **3,** 10-16, (2013)

[20] A.J.M. Jawad, M.D. Petkovic and A. Biswas, "Modified simple equation method for nonlinear

evolution equations", Appl. Math. Comput**., 217,** 869-877, (2010)
[21] J.H. He and X.H. Wu, "Exp-function method for nonlinear wave equations", Chaos, Solitons Fract., **30,** 700-708, (2006)
[22] A. M. Wazwaz, "A sine-cosine method for handle nonlinear wave equations", Applied Mathematics and Computer Modeling, **40,** 499-508, (2004)
[23] D. Liu, "Jacobi elliptic function solutions for two variant Boussinesq equations", Chaos solitons Fractals, **24,** 1373-85, (2005)
[24] Y. Chen and Q. Wang, "Extended Jacobi elliptic function rational expansion method and abundant families of Jacobi elliptic functions solutions to (1+1)-dimensional dispersive long wave equation", *Chaos solitons Fractals*, **24**, 745-57, (2005)
[25] M.L.wang and Y.B.Zhou, "The periodic wave solutions for the Klein-Gordon-Schrodinger equations", *Phys.Lett.A*, **318**, 84-92, (2003)
[26] M.L.Wang and X.z.Li, "Extended F-expansion method and periodic wave solutions for the generalized Zakharov equations", *Phys.Lett.A*, **343**, 48-54, (2005)
[27] M.R. Miura, "Backlund transformation", Springer, Berlin, (1978)
[28] V.B. Matveev and M.A. Salle, "Darboux transformation and solitons", Springer, Berlin, (1991)
[29] M. Wang, "Solitary wave solutions for variant Boussinesq equations", *Phy. Lett. A*, **199**, 169-172, (1995)
[30] E.M.E. Zayed, H.A. Zedan and K.A. Gepreel, "On the solitary wave solutions for nonlinear Hirota-Sasuma coupled KDV equations", Chaos, Solitons and Fractals, 22, 285-303, (2004)
[31] M.L. Wang, "Exact solutions for a compound KdV-Burgers equation", Phys. Lett.A, 213, 279-287, (1996)
[32] G. Adomain, "Solving frontier problems of physics: The decomposition method", Kluwer Academic Publishers, Boston, (1994)
[33] A.M. Wazwaz, "Partial Differential equations: Method and Applications", Taylor and Francis, (2002)
[34] Sirendaoreji,J.Sun, "Auxiliary equation method for solving nonlinear partial differential equations", Phys.Lett.A, 309, 387-396, (2003)
[35] Sirendaoreji, "Auxiliary equation method and new solutions of Klein-Gordon equations", Chaos Solitions Fractals, 31, 943-950, (2007)
[36] M.M. Zhao and C. Li, "The $\exp(-\varphi(\eta))$-expansion method applied to nonlinear evolution equations", http://www. Paper. Edu. Cn
[37] G.Tzitzeica, "Geometric infinitesimale-surune nouvelle classe de surface", C. R. Math. Acad. Sci. Paris, 150, 227, (1910)
[38] A.M. Wazwaz, "The tanh method:solitons and periodic solutions for the Dodd-Bullough-Mikhailov and the Tzitzeica-Dodd-Bullough equations", Chaos Solitons Fractals, 25(1), 55-63, (2005)
[39] J. H. He and M. A. Abdou, "New periodic solutions for nonlinear evolution equations using exp-function method", Chaos Solitons Fractals, 34, 1421-1429, (2007)