Design of Virtual Objects Using Transformation Optics

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Abstract

Two structures of virtual targets filled with metamaterials are investigated through transformation optics to tailor the specific electromagnetic fields into desired spatial patterns. One virtual structure is a square column object transformed from a dielectric cylinder and the other virtual structure is a cylinder object transformed from a dielectric square column. Because the electromagnetic parameters in the virtual objects are obtained from real objects by the method of transformation optics, the scattering fields of virtual structures are the same as those of the real objects. The numerical simulations further prove the correction of theoretical results.

***Keywords*: Scattering fields, Transformation optics, Illusion optics**

1. Introduction

Recent years, many people give their vote to the metamaterial because of its novel electromagnetic properties. Especially, since Pendry [1] first proposed the concept of transformation optics [2-3] that can control electromagnetic field by the designers' intention through coordinate transformation method, a load of applications have been emerged sequentially, such as the cloak [4-6], the rotating [7] and the splitting[8]. One of the most excellent applications of transformation optics is illusion optics which represents an image of a real object in the world perceived by the eyes that is deceptive or misleading [9-12]. However, illusion optics proposed first by Lai et al [12] need to be implemented by complementary medium which is left-handed material with double negative parameters (negative permittivity and negative permeability) [13]. However, the absence of metamaterial for its negative permittivity and negative permeability property makes it being produced only in artificial method. Although some novel manufacture methods of left-handed material have been proposed, it is widely known that it has a lot of difficulties in the fabrication process [14]. Hence, such illusion devices are highly demanded on the material properties, and extremely difficult to be realized. In order to manufacture the illusion optics devices easily in reality, Jiang et al. [15-18] have proposed the other illusion optics devices composed of metmaterials based on transformation optics. In such illusion media, all components of constitutive parameters in the principle coordinate system are finite and positive and they could be realizable using artificial structures.

In this paper, we focus on the illusion optics which contains the finite and positive metamaterials. Here, we discuss both cases of transformation shapes. First, we transform a two-dimension (2D) dielectric cylinder into a 2D square column using coordinate transformation. Based on the method of transformation optics, the scattering fields of transformed square column are consisted of those of a untransformed dielectric cylinder. The second, in contrast to the first case, a 2D dielectric square column is converted into a 2D dielectric cylinder. Then, it is similar to the first results, in such transformation, the scattered fields of transformed cylinder and those of untransformed square column are identical. In other words, the first case illustrates that the scattered fields of a two dimension square column can be replaced by those of a two-dimension dielectric cylinder, and the second case implies that the scattered fields of circular can be substituted for that of two-dimension square column. Hence, our illusion devices can mislead the detection of radar to realize the aims of electromagnetic stealth in military. Our paper is arranged as follows: Firstly, a 2D cylinder and square column are transformed into a 2D square column and cylinder, respectively, and anisotropic electromagnetic parameters of transformed regions can also be obtained based on the method of transformation optics. Secondly, the distributions of scatting fields are numerical simulated before transformation and after.

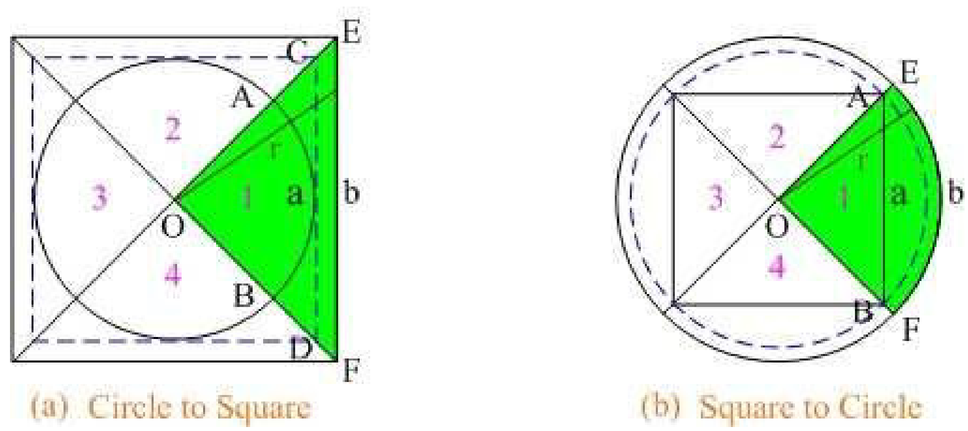

Figure 1: (a): A dielectric cylinder of radius '*a*' is transformed into a square column with side *b*, which is divided into 1, 2, 3, 4 four sub-domain. (b): A dielectric square column with side '*a*' is transformed into a cylinder of radius *b*, which is divided into 1, 2, 3, 4 four sub-domain.

2. Theoretical Analysis

The proposed structures are schematically shown in Fig. 1. Fig. 1(a) represents the first structure which conclude a 2D square column with the length of the side r = *b* inside of a 2D cylinder with radius of r = *a*. Our aim is to remain the same external scattering field through the method of transformation optics, when a cylinder with radius *a* is transformed into a square column with the length of the side r = *b*. The blue dotted line illustrated the shape after transformed in Fig 1(a). In contrast to Fig 1(a), Fig. 1(b) is the second planned structure, where a square with a side length a circumscribed a 2-D cylinder with radius of *b*. Similarly, our propose is to change the square column into a cylinder, and the same time their external scattering fields before and after transformation are consistent.

For the first structure (Fig.1a), the whole region is divided into four sub-areas, named as 1, 2, 3, 4 respectively. For simplicity, we take the first sub-area as example to illustrate how to transform a square column into a cylinder in the cylindrical coordinates system. We adopt a two-step transformation to complete the entire process. First, the irregular shape AEFB is compressed into an isosceles trapezoid CEFD which is necessary to reserve space for the next transformation. The transformation relationship of this step is expressed as follows:

$$r' = \frac{b-a}{b-a\cos\theta} r + \frac{ab(1-\cos\theta)}{\cos\theta(b-a\cos\theta)} \quad ; \quad \theta' = \theta \quad ; \quad z' = z \tag{1}$$

Second, the sector OAB is stretched into an isosceles triangle. The transformation expressions of the second step are:

$$r' = \frac{1}{\cos\theta} r \quad ; \quad \theta' = \theta \quad ; \quad z' = z \tag{2}$$

where *r', θ', z'* represent new coordinates (transformed coordinate) and *r, θ, z* are original coordinates.

Taking advantage of method of reference [1, 2], the permittivity and permeability tensors of the transformation region are calculated by

$$\overline{\varepsilon}' = \Lambda \varepsilon \Lambda^T / \det(\Lambda) \quad ; \quad \overline{\mu}' = \Lambda \mu \Lambda^T / \det(\Lambda) \tag{3}$$

in which (ε, μ) and ($\overline{\varepsilon}, \overline{\mu}$) are the constitutive tensors in the original region and the transformation region, respectively, and Λ is the Jacobian transformation matrix with components $\Lambda_{ij} = \partial x_i / \partial x_j$, corresponding to the mapping from the original region to the transformation region, det(Λ)is the determinant of the matrix of Λ. Based on transformation optic, the original regions are distorted and stretched, and not folded, it is obviously that the electromagnetic parameters are anisotropic, positive and gradual in the transformation regions. Therefore, those special electromagnetic parameters can be fabricated and realized by the metamaterials in practice.

Other three sub-areas have the same transformation method as the first sub-area. The corresponding transformation formulas for the 2, 3, 4 areas can be readily obtained by applying rotation operators with rotation angles of (π/2), π and (3π/2) around the z-axis to equation (1) and (2). Hence, after transformation, cylinder in region can be converted into square.

There is the same method to analyze for the second structure in the cylindrical coordinates. Likely, the whole region is divided into four sub-areas, named as 1, 2, 3, 4 respectively. For simplicity, we only illustrate the transformation equations for the first sub-area. 2, 3, 4 three sub-areas have similar transformation way as that of the first sub-area, except that the angle θ need to be changed into θ + (π/2), θ + π, θ + (3π/2) respectively, and others are not changed. Likewise, the transformations of the first sub-area conclude two steps. First step, the irregular shape AEFB is changed into regular fan ring AEFB. Second step, the isosceles triangle OAB is stretched into a sector OAB. The transformation equations both steps are given by:

$$r' = \frac{(a-b)\cos\theta}{a - b\cos\theta} r + \frac{ab - ab\cos\theta}{a - b\cos\theta} \quad ; \quad \theta' = \theta \quad ; \quad z' = z \tag{4}$$

$$r' = r\cos\theta \quad ; \quad \theta' = \theta \quad ; \quad z' = z \tag{5}$$

Then the same reason, after getting all four sub-areas transformation expressions, we can obtain the expressions of electromagnetic parameters according to reference [1, 2]. Finally, the square column can be transformed into cylinder softly with external scattering field unchanged case which misleads the detector that the circular object instead of square object.

3. Numerical simulations

In order to prove the correctness of above theoretical analysis, a finite element method has been carried out to demonstrate the function of illusion optics through numerical simulation. We select the computed range x = −2.5 ~ 2.5m, y = −2 ~ 2m, where a 2D cylinder of radius a = 0.4m is located at the origin. Assuming a TE plane wave with frequency *f* = 1Ghz is incident from left boundary, Fig.2(a) shows the distribution of electric fields in the X-Y plane, where a medium cylinder with $\varepsilon_r = 4$ and $\mu_r = 1$. According to formulas (1,2), we can change a cylinder of radius *a* = 0.4m into a square column with side *b* = 0.5m after two transformation. If the transformation region (2D square column) is filled with gradually anisotropic metamaterials based on formula (3), the electric fields along X-Y plane are illustrated in Fig.2(b). Comparing with both pictures, we can see that both electric fields are exactly the same outside of the transformed region. This shows that the external scattering fields remain unchanged when the cylinder is transformed into the square column. That is to say, the scattering field of a square column object with special metamaterials is the same as those of a cylinder object. Therefore, a square column object can be regarded as an illusion object, which can mislead the detector, and the real target object can be hided.

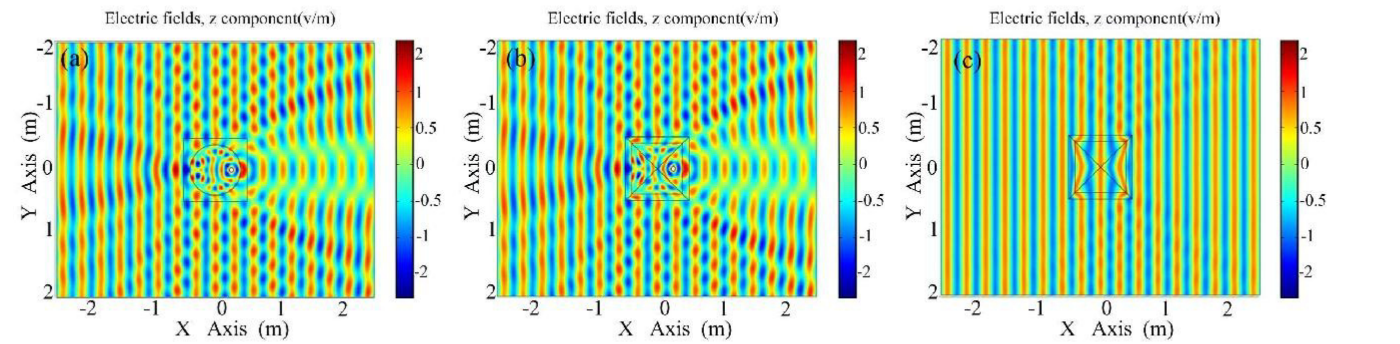

Figure 2: The distributions of electric fields of dielectric cylinder and the corresponding illusion device with a = 0.4, b = 0.5, f = 1Ghz. The computed range x from −2.5 to 2.5, y from −2 to 2. (a) The dielectric cylinder. (b) The corresponding illusion device. (c) : The transparent medium in the illusion device.

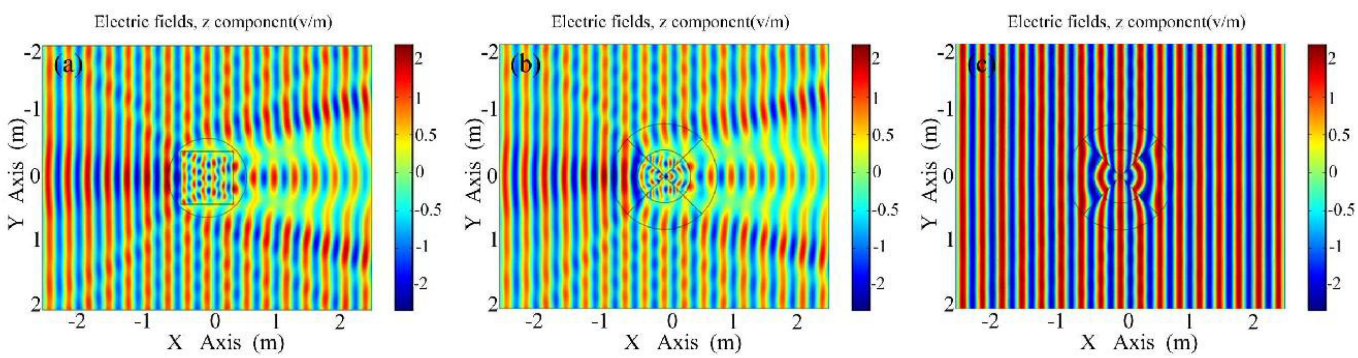

Figure 3: The distributions of electric fields of dielectric square and the corresponding illusion device with a = 0.4, b = 0.6, f = 1Ghz. The computed range x from −2.5 to 2.5, y from −2 to 2. (a) The dielectric square column. (b) The corresponding illusion device. (c) : The transparent medium in the illusion device.

If the media in the cylinder is filled with transparent media (such as vacuum $\varepsilon_r = 1$ and $\mu_r = 1$), a incident plane wave remains characters of plane wave after transmits through the transparent object. If the transparent cylinder is changed into a square column according to the above transformation, the electric field outside transformation region should also be plane wave. Fig.2(c) shows the distribution of electric fields along X-Y plane, where a plane wave is incident on a square column of side b = 0.5m transformed from a transparent cylinder of radius a = 0.4m. The figure shows that the distribution of electric fields is indeed plane waves, which prove once again that the theoretical analysis is correct. On the other hand, if the center region of transparent cylinder remains unchanged, a transparent cylinder can become a square radome by a similar transformation, which is the same as the results of reference [19].

For the second structure, we choose the same computed region and incident plane wave as Fig. 2. In this case, a square medium column of side a = 0.4m with $\varepsilon_r = 4$ and $\mu_r = 1$ is transformed into a cylinder of radius b = 0.6m. Fig.3(a, b) shows the distributions of electric fields along X-Y plane, when a plane wave is incident on the square column and cylinder transformed from square column, respectively. Although the distributions of electric fields in the square column and cylinder are different, the external electric fields outside the transformed region are consistent, which means the scattered fields produced by cylinder object filled with anisotropic metamaterials are the same as those of a dielectric square column. Hence, the cylinder object can be regarded as an illusion object of a square column, which makes radar perplex real target from false target. At last, if the square column is filled with transparent medium, the scattered fields outside transformation region are still plane waves. The simulation is depicted in the (Fig.3(c)), which is similar to the case of Fig.2(c).

4. Conclusion

In conclusion, we investigate the two structures of illusion optics to achieve the purpose of virtual target. One case is the scattering fields of a real dielectric cylinder that is the same as those of a virtual square column, when the dielectric cylinder is transformed into the dielectric square column based on transformation optics. The other case is a real dielectric square column transformed into a virtual dielectric cylinder, and the scattering field of square also remains the same. The numerical results prove that two structures of illusion optics can be realized by transformation optics. Because all electromagnetic parameters in the virtual objects are anisotropic and positive, it is not difficult to synthesize these devices by metamaterials.

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