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Modal interpretation of Quantum mechanics provides the framework for the Description of physical reality

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Abstract

The question whether the quantum description of physical reality may be considered complete remains a challenge among a number of physicists even today. These challenges have their origins from the quantum measurement problem.

This paper tries to address this problem by focusing on the quantum logical perspective and particularly on how modal interpretation of quantum mechanics leads to the contextual description of physical reality. It argues that in classical mechanics any composite system can be completely described by the state of its parts. But in quantum mechanics the composite system can be in the entangled state and therefore cannot be described by giving a pure state for each subsystem.

By using the modal interpretation of quantum mechanics, we are led to the description of reality that cannot be postulated rather to the reality in postulation. Reality in postulation is contextual. In this sense transition from postulation of reality to reality in postulation is analogously similar to transition from Boolean structure to non – Boolean structure of the universe, from the localized systems to non – localized systems.

***Keywords*:** Modal interpretation, Quantum logic, measurement problem, orthomodular lattices, contextual and non – locality.

1. Introduction

The main task of physicists is to provide the understanding of physical reality that is presumed to underlie the observed results. In physics the description of physical reality is directly associated with observations and measurements. In classical physics the description of physical reality was thought to be obvious and natural. Since the beginning of quantum mechanics, the classical physics description of physical reality has faced a lot of challenges. The question whether the quantum description of physical reality may be considered complete remains a challenge among a number of physicists even today. These challenges have their origins from the quantum measurement problem.

Quantum measurement problem arises from the fact that in classical mechanics any composite system can be completely described by the state of its parts. But in quantum mechanics the complete description of the composite system is not possible. In quantum mechanics the composite system can be in the entangled state and therefore cannot be described by giving a pure state for each subsystem. Various attempts have been made to resolve the problem of measurement in quantum mechanics. Some of the main attempts are the Copenhagen interpretation, von Neumann, Schrodinger's cat, hidden variables, statistical interpretation, many worlds and quantum logic. Attempts to solve the measurement problems have led to a number of competing theories of the description of physical reality. Up to now, there is not one well defined theory of the description of physical reality which is accepted by all the physicists. This means that the problem still persists. This article is another attempt and it focuses on the quantum logical perspective and particularly on how modal interpretation of quantum mechanics leads to the contextual description of physical reality.

2. Challenges of the Description of Physical Reality: the logical perspective

Classical physics depends entirely on observation and measurements in understanding of physical events. Measurements in classical physics provide information on the properties (states) of observed results. Researches that have been done since the works of von Neumann have led to an interesting connection between logic, geometry and probability theory. The works of von Neumann in particular have revealed that the states of physical events in classical physics are associated with Boolean structures[1].

Measurements in classical physics are based on the Boolean structures. Boolean structures are based on the law of the excluded middle which asserts that, a proposition must be either *true* of *false*. We can say that the foundations of the classical description of physical reality have their roots in the Aristotelian logic and Euclidean geometrical models.

Einstein, Podolsky and Rosen were the first physicists to question whether the quantum mechanical description of reality could be considered to be complete[2].They came up with what is known as EPR thought experiment. The Einstein-Podolsky-Rosen (EPR) thought experiment argues that the description of physical reality provided by quantum mechanics is incomplete. Deep down the logic behind the EPR description of physical reality is the logical law of excluded middle. According to EPR"if, without in any way disturbing a system, we can predict (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of the physical reality corresponding to this physical quantity." We observe that EPR starts with the conviction that there is an *objective reality*; whether we observe it or not. Physical reality according to EPR does not depend on observation or measurements, but it exists independently and separately from our observations. Therefore, measurements must be able to predict with certainty the values of the physical quantities.

As we have mentioned before, the classical physics model which has been the basis of our understanding of physical reality is based on Boolean structures and logic. In this model, matter was composed of distinct elements that move in accordance with deterministic mathematical rules. Each element for example had definite position and velocity. The success of Newtonian mechanics was due these simplified concepts which were natural and obvious to our intuitions. Moreover, classical physics has imposed some constants of motion and constraints which reduce the description of motion to a simple set of differential equations: a rigid body is an example of such constraints.

The classical physics description of physical reality has faced a lot of challenges since the beginning of quantum mechanics. At the heart of these challenges are the differences in the logic of things that lead to what is familiarly known as the quantum measurement problem. The measurement problem arises from the fact that measurement "at the microscopic level does not allow definite outcomes to be realized, whereas at the level of our human consciousness it seems a matter of direct experience that such outcomes occur.[3]" In a Young's slits experiment for example, quantum mechanical description of the ensemble is by a superposition of amplitudes corresponding to alternative microscopic possibilities A and B (e.g., A went through slit 1 and B went through slit 2). The quantum logic interpretation proposes to solve the measurement problem with the simple postulate that quantum logic is the logic of the world. [4]

3. Modal Interpretations of Quantum Mechanics

In traditional approaches to quantum measurement theory, projection postulate played a predominant role. Projection postulates asserts that for a closed system with Hamiltonian **H** the state vector evolves continuously in accordance with the Schrodinger equation,

$\boldsymbol{H|\psi\rangle = i\hbar\partial|\psi\rangle/\partial t}$. But when a measurement of the dynamical variable Q is performed on the system, the state vector is said to collapse to an eigen – vector $|\boldsymbol{q_i}\rangle$ corresponding to the observed eigenvalue $\boldsymbol{q_i}$, $\mathbf{Q}|\boldsymbol{q_i}\rangle = \boldsymbol{q_i}|\boldsymbol{q_i}\rangle$. In other words the projection postulate asserts that the state of the physical system collapses onto a state corresponding to the value found in the measurement. Collapse of the state is generally abrupt, discontinuous, and stochastic [5]. However, this postulate leads to many difficulties [6]. In fact the view that the collapse of a quantum state is a physical process arises from a misconception of probability and the role it plays in quantum mechanics.

In order to solve problems arising from the projection postulate, a number of interpretations were proposed. One of the interpretations is the class of modal interpretations. Modal interpretation is a non-relativistic quantum theory which encompasses a class of interpretations. It is a non – collapse interpretation, where the quantum state of a system describes its possible properties rather than the properties that it actually possesses. Modal interpretations intend to provide, for every instant, a set of definite-valued properties and their probabilities.

The fundamental difference with the modal interpretation is that the change in the quantum state $|\psi\rangle$ manifests itself directly at a modal level, the level of possibility rather than actuality through the determinate sub-lattice defined by $|\psi\rangle$ and the position in configuration space as the preferred determinate observable.

In this article, we are focusing on the Van Fraassen's proposal of modal Hamiltonian interpretation which proposes a distinction between what he called the *dynamical state* and *the value state* of a system at any instant. According to Van Fraassen, the dynamical state is represented by our familiar vector or density matrix in Hilbert space which refers to the state which physical properties of the system *may possess*, and *properties that the system may have* at later times. In other words, the *dynamical state* determines what may be the case. The *value state* on the other hand, is quite different from the *dynamical state*. The *value state* represents what actually is the case, that is, all the system's physical properties that are sharply defined at the instant in question.

The general idea of Van Fraassen proposal is that physical systems *at all times possess* a number of well-defined physical properties which are definite values of physical quantities. These properties can be represented by the system's value state and they may change in time. The change of properties in time may be due to a change in size, structure or composition.

According to Van Fraassen, given a system at a given time, the possible "value states" follows the following restriction: "propositions about a physical system cannot be jointly true, unless they are represented by commuting observables."[7] In other words, the non-commutativity of observables imposes limits not on our knowledge about the properties of a system, but rather on the possibility of joint existence of properties, independently of our knowledge. For example, non-commuting quantities, like position and momentum, cannot jointly be well-defined quantities of a physical system.

Therefore, in these cases one would expect the dynamical state to generate a probability measure over the set of possible measurement results. Indeed, *the dynamical state in general only tells us what is possible*. The dynamical state possibilities provide a precise criterion for the preferred factorization of the Hilbert space into factors representing elemental systems.

3.1.Essential features of modal interpretations

In spite of the differences among them, all the modal interpretations agree on the following points:

- The interpretation is a non – collapse interpretation where the quantum state of a system describes its possible properties rather than the properties that it actually possesses.
- The interpretation assumes that existence of a special set of disjoint systems that fixes the preferred factorization of the Hilbert space.
- The interpretation assumes that *quantum systems always possess a number of definite properties, which may change with time.*
- The dynamical state of the system (pure or mixed) tells us what the *possible properties of the system and their corresponding probabilities are.*
- The dynamical state always evolves unitarily according to the Schrödinger equation.

3.2.The interpretation of probability in Modal interpretation

We know that in quantum mechanics the description of physical system is based on the notion of *probability*. The recent understanding of probability is stimulated by the appreciation of the relationship between Boolean algebra, set algebra and logic. The recent description of physical reality is stimulated by the more recent developments in our understanding of quantum probability. The connection between logic, geometry and probability theory has led to quantum mechanics description of a physical system in terms of states. A state is a compendium of probabilities of all possible answers to all possible questions one can ask of the system. Therefore, there is no other deeper underlying theory that gives a fuller description of physical reality[5].

However, we observe that the Born probability which is defined over projector operators on a Hilbert space does not satisfy the definition of probability of Kolmogorov which applies to a Boolean algebra of events. Since events of quantum systems are non – Boolean, we require a new understanding of the notion of probability. The new understanding of quantum probability uses the algebraic account of quantum theory which makes use of Boolean algebra, partial Boolean algebra and orthomodular lattices.

In the axiomatic approach of von Neumann projection operators play a key role. The spectral decomposition theorem [8] allows to associate a projection valued measure to any quantum observable represented by a self adjoint operator[9]. It turns out that the set of projection operators can be endowed with a lattice structure; more specifically, the form an orthomodular lattice [10]

These structures can be found embedded in Hilbert spaces, which are the complex topological vector spaces appropriate to quantum mechanics. According to Oliver Brunet, orthomodular lattices provide us with a better way to describe the state of quantum systems, based on finite measurements because orthomodular lattices constitute a more general algebraic formalism.

In orthomodular lattices the quantum state (given a feasible measurement) indicates some measurement outcomes which are known to be impossible with probability = 0 and others with probability that is non – zero, which provides as much information as possible.

> “Not only quantum states cannot be regarded as partial states, but partial states are infinitely more informative than quantum states.” [11]

This means that;

> “A partial description provides information about which outcomes will not occur, and not about which outcomes will.” [11]

Therefore, in quantum mechanics we use non – Boolean orthomodular description which chooses a subset of $\mathcal{L}_x$ to be the domain of the quantum probability measure [4]. In this description the two-valued logic, which is inadequate to deal with the complexity of the real world, is considered to be the particular case of the formalism of quantum mechanics. This means that the projectors of two – valued logic are the extreme cases of the quantum systems. Therefore, the logic of quantum mechanics not only, has all values between **0** and **1** but also considers **0** and **1** as limiting cases.

Quantum logic in orthomodular lattices gives rise in a natural way to some kind of modalities. These modalities are the ones which inspired the modal interpretation of quantum mechanics. Modal interpretations have a straightforward way out of the measurement problem because it asserts that, "although only one state of affairs is *actual*, the total state describes all *possibilities* – it gives rise to a probability distribution that comprises both the *actual* and the *possible*." [12]

The central idea of modal interpretation in general is that the physical systems at all times posses a number of well – defined *physical properties which change in time*. This means that the change of a physical system in time is described by a change of its properties. The dynamical state $|\boldsymbol{\psi}\rangle$ determines the set of possible value states and their possible evolution in time. Therefore quantum properties take the form "the observable $\mathbf{A}$ has value $\boldsymbol{a}$, and every such property is associated with a closed subspace of a Hilbert space, or equivalently with the projection on their subspace." [13]

The statement '$\mathbf{Q}$ has some definite value' is equivalent to '$\mathbf{A}$ has the value $\boldsymbol{q_1}$, or $\mathbf{Q}$ has the value $\boldsymbol{q_2}$,……'. (There is one disjunct for each eigenvalue of **Q**). The quantum – logical representation of this statement is:

$$\boldsymbol{P_{q1}^{Q}} \vee \boldsymbol{P_{q2}^{Q}} \vee \ldots..$$

Or

$$\bigvee_{i} \boldsymbol{P_{qi}^{Q}}$$

Since the eigenspace of any operator span the entire Hilbert space, therefore:

$$\bigvee_{i} \boldsymbol{P_{qi}^{Q}} = \mathbf{1} \ \ which\ is\ a\ tautaulogy$$

We must not say for example 'the observable Q, P,..jointly have some set of definite values'.

$$\bigvee_{i,j,\ldots} \left(\boldsymbol{P_{qi}^{Q}} \wedge \boldsymbol{P_{pj}^{P}} \wedge \ldots\right)$$

In lattice theory, this expression is zero subspace, which always has probability $\mathbf{0}$ and therefore corresponds to the always – *false proposition*.

Tautology is the always – true proposition in contrast with *contradiction* which is the always – false proposition. This means that the statement '$\mathbf{Q}$ has some definite value' is a tautology in quantum logic that is every observable has a definite value, according to the quantum logic interpretation[4]. This implies that, at any time $\boldsymbol{t}$, there exists a definite momentum possessed by a system at $\boldsymbol{t}$ and there exists a definite position possessed by a system at $\boldsymbol{t}$, and yet assent to the quantum mechanical theorem that the system does not possess a definite position and a definite momentum at the same time $\boldsymbol{t}$, indicates that we will still have unlimited number of observables.

According to modal interpretations, this implies that prior to observation quantum objects exists in all possible states and it is not possible to assign at the same time definite values to certain pairs of physical quantities. Now, in order to explain logically observables, let us "pretend that all physical magnitudes have *finitely* many values, instead of *continuously many*" [14]. Putnam says that suppose that the admissible values of position and momentum for a system **S** are, $\boldsymbol{p_1}, \boldsymbol{p_2}, \ldots \boldsymbol{p_n}$ and $\boldsymbol{q_1}, \boldsymbol{q_2}, \ldots \boldsymbol{q_n}$ respectively.

Let $\boldsymbol{P_i}(\boldsymbol{Q_i})$ denote the subspace of $\boldsymbol{\mathcal{H}_s}$ spanned by the proper vectors of position (momentum) corresponding to $\boldsymbol{p_i}(\boldsymbol{q_i})$**.**

Write $\boldsymbol{p_i(t)}$ and $\boldsymbol{q_i(t)}$ for "**S** has position $\boldsymbol{p_i}$ at time $\boldsymbol{t}$" and "**S** has momentum $\boldsymbol{q_i}$ at time $\boldsymbol{t}$", respectively.

Then, if it is verified that **S** has position $\boldsymbol{p_k}$ at time $\boldsymbol{t}$, we may confidently assert that:

$$\boldsymbol{p_k(t)} \wedge \left(\boldsymbol{q_1(t)} \vee \ldots \vee \boldsymbol{q_n(t)}\right)$$

But certainly we cannot assert;

$$\left(\boldsymbol{p_k(t)} \wedge \boldsymbol{q_1(t)}\right) \vee \ldots \vee \left(\boldsymbol{p_k(t)} \wedge \boldsymbol{q_n(t)}\right)$$

This is due to the fact that the distributive laws are not valid in quantum logic.

The failure of distributive laws in quantum mechanics is the one which lead us to the modal interpretation of quantum mechanics.

Modal interpretation is the partial description of quantum state and it allows us to categorize "our experiences as being more or less *deep*, more or less *rea*l, along a continuum, or dimension, or realness. This dimension is quite different from the dimensions of space and time." [15] We can easily observe that when one moves into a depth or a height of experience, one moves away from time into an extra – temporal dimension. The description of quantum states in terms of orthomodular lattices represents the states in quantum mechanics by means of the *space of possible states*.

4. Contextual Description of Quantum Reality

Bub's concept of the universe as a non – Boolean structure that changes dynamically provides us with the foundations for the description of physical reality in physics. The physical reality in this context is no longer defined in terms of space and time but rather in terms of possibility structure of our universe.

The description of physical reality arising from quantum mechanics is *contextual* and somehow differs from classical physics description. Contextuality is a quantum phenomenon which was demonstrated by the Bell-Kochen-Specker theorem. The theorem implies that the measurement result of quantum observable is dependent upon which other commuting observables are within the same measurement set.

The modal ideas lead us to the contextual description of physical reality. It is contextual in the sense that it can never be understood rationally in terms of a unique and unambiguous model but rather in terms of multiple models. The same reality may be described in various ways; physical, chemical, biological, mathematical or even social. The contextual description of quantum reality is inspired by the quantum state vector which postulates that: quantum objects prior to observation exist in all possible states.

In terms of modal interpretation reality cannot be postulated rather reality is *in postulation*. Reality is whatever the wave function $\psi(r,t)$ is capable of describing. *Reality in postulation* is contextual and quantum logic is the logic of the reality in postulation. In this sense we have the same mixed state ρ, the same mixture, and different blends for it, each of them offering a different aspect of reality.[16] In this sense transition from postulation of reality to reality in postulation is analogously similar to transition from Boolean structure to non – Boolean structure of the universe, from the localized systems to non – localized systems.

The modal interpretation of quantum mechanics has broadened our concept of reality to include the reality beyond our observation. With modal interpretation, we have come to realize that reality may be real without being a concrete thing; being a concrete thing is only one of the modes of being of a reality. For example, electromagnetic theory tells us that colors are the result of photons of a particular energy affecting us. But we know that there are two realities (the photons and the colors); the colors are various modes of perception of the photons as waves and particles are various modes of perception of the reality of light. Therefore, reality may exist in multiple types and at various levels. The multiplicity of representation is not just imposed by external instrument but truly intrinsic to the physical reality, even when it is unobserved. The multiple representations are represented by the range of degree of probability by a pure or mixed quantum state and therefore, it becomes essential for speaking about reality. “In our description of nature, the purpose is not to disclose the real essence of the phenomena but only to track down, so far as it is possible, relations between the manifold aspects of our experience. [8]” Partial description of quantum state allows us to categorize “our experiences as being more or less *deep*, more or less *rea*l, along a continuum, or dimension, or realness. This dimension is quite different from the dimensions of space and time.” We can easily observe that when one moves into a depth or a height of experience, one moves away from time into an extra – temporal dimension.

5. Conclusion

In this article we have argued that the quantum state vector that is essential for the description of quantum reality;

- ❖ Provides us with a way of multiple presentation of reality. Therefore, quantum reality may be described by many possible ways, all mutually exclusive, without leading to paradoxes or internal contradictions.
- ❖ Has made us realize that nature does not favor any specific model when we are not observing it; rather it is a mixture of the many possibilities.
- ❖ Leads us to transition from postulation of reality to reality in postulation.
- ❖ Classical physics is not our unique reference towards the knowledge of reality where logic can be applied and through which we can legitimately speak. From the modal interpretations of quantum mechanics, we conclude that quantum reality can never be described nor understood rationally in terms of a unique and unambiguous model. Quantum reality requires and accommodates various possible models. This conclusion flown naturally from the quantum state vector since quantum reality is whatever the quantum state vector is capable of describing.

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