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Confinement and Asymptotic Freedom in a Purely Geometric Framework

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Abstract

The most important properties of strong interactions, confinement and asymptotic freedom, can be explained in a purely geometric way, using a non-local modification of the general relativity. At the same time, the dichotomy matter-field of the Einstein equation is eliminated and the physical world is described only by means of a unified field. Hadrons can be identified with "strong" black holes. The uncertainty principle emerges naturally in this model as consequence of the non-local modification of the General Relativity.

Keywords: confinement, asymptotic freedom, nonlocality, General Relativity, Heisenberg principle.

1. Introduction

In a previous paper [1], we have developed a non-local theory based upon a modification of the general relativity. We review briefly the main findings of the paper.

It is well known that in the general relativity, the theory nonlinearity implies that the gravitational field can become source of itself. However, in the Einstein equation,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\chi T_{\mu\nu}, \qquad R = g_{\mu\nu} R^{\mu\nu}, \tag{1.1}$$

where $R_{\mu\nu}$ is the Ricci tensor, $T_{\mu\nu}$ the energy-momentum tensor, χ a coupling constant and $g_{\mu\nu}$ the metric tensor, matter is always present through the energy-momentum tensor, that can be calculated only on a phenomenological basis.

In a nonlinear field theory, it is possible to eliminate the matter, because a particle becomes a small spatial region, where the field assumes particularly high (eventually infinite) values. A particle is a lump of energy, a nonlinear field excitation that is able to maintain its identity during the time. All the attempts carried out by Einstein in order to construct a nonlinear field theory can be really considered as models where particles would be particular non singular solutions of the field equation.

In conclusion, the Einstein theory is characterized by the spacetime deformations expressed with the Ricci tensor, but the nonlinearity of the LHS of (1.1) is "spoilt" by the RHS, where the energy-momentum tensor is simply the source term of the linear field equations.

However, we note that relativistic quantum field theory, that is produced by the fusion of quantum mechanics with special relativity, reduces the field to a collection of particles (the quanta of the field) by means of the second quantization method. That is the contrary of Einstein's ideas about the superiority of the field with respect to the matter.

To avoid misunderstandings, it is necessary to point out that the Einstein's opposition to the nonlocality [2] refers to particle interactions. The quantum mechanics incompleteness would be caused by absurd nonlocal (faster than light) effects that link together a couple of particles, that have interacted in the remote past. In the classical field theories, particles exist independently from the field, that is the force mediator among particles. In this framework, it is impossible to conceive an instantaneous propagation of the forces.

However, if matter is not independent from the field but is a part of the field itself, then the nonlocality has a completely different meaning, because it is the expression of the coherence of the unified field.

We have identified four postulates for the new theory:

i) Covariance principle: all the physical laws must be independent from the reference frame, i.e they must possess the same form in all the reference frames and that implies their tensorial character. Physical laws must be invariant not only for all the physically feasible reference frames, but also for a generic coordinate change that produces a not feasible reference frame (for example the Schwarzschild reference frame);

ii) Equivalence principle: the field can be always eliminated by an appropriate choice of the reference frame; in other words, mass and charge are equal;

iii) Uniqueness principle: the only reality is the (unified) field and particles are not irreducible entities. They are a part of the field, produced by its nonlinear structure;

iv) Nonlocality principle: the field equation must produce nonlocal correlation effects.

The first requirement is similar to the corresponding general relativity principle, the second extends to all interactions the general relativity equivalence principle, the third is the Einstein's dream, while the fourth would imply the temporal irreversibility.

We are then induced to postulate the following field equations

$$R_{\mu\nu;o} = 0, \tag{1.2}$$

i.e. the covariant derivative of the Ricci tensor must vanish; in this way we can determine the spacetime metric structure (as usual comma denotes differentiation).

As usual $g_{\mu\nu}$ is the metric tensor, with

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \, . \tag{1.3}$$

Due to the symmetry of the Ricci tensor, the system (1.2) corresponds to 40 independent equations. The system (1.2) can also be written in the form

$$R_{\mu\nu,o} - \Gamma^{\tau}_{o\mu} R_{\tau\nu} - \Gamma^{\tau}_{o\nu} R_{\tau\mu} = 0 . \qquad (1.4)$$

Note that the field equations (1.4) are linear with respect to the third order derivatives. As a consequence (1.4) is not invariant to the substitution *t* with –*t* and, besides, the presence of third order derivatives imply nonlocal effects.

We have considered the Schwarzschild problem (the spherical stationary case) where the metric tensor is

$$ds^2 = \alpha c^2 dt^2 - \left[\frac{1}{\alpha} dr^2 + r^2 (\sin^2 \vartheta \, d\phi^2 + d\vartheta^2) \right], \qquad (1.5)$$

where $\alpha = \alpha(R)$.

The field equations (1.4) can be exactly resolved in the spherical stationary case and for the metric tensor we find [1]

$$\alpha = 1 + KR^2 + \frac{A}{R}, \qquad (1.6)$$

where *A* and *K* are appropriate integration constants. Note that this solution is formally equal to that obtained with the Einstein equation with the cosmological term, but in the latter case *K* is not an integration constant, while now *K* can assume different values depending on the considered problem.

It is well known that in general relativity the motion of a particle under the action of the gravitational field obeys the geodesic equation

$$\frac{d^2 x^\mu}{ds^2} + \Gamma^{\mu}_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0 \ . \qquad (1.7)$$

Equation (1.7) is not valid in the new theory, because a particle is not independent and separated from the field. However, in many cases we assume the validity of (1.7) in the first order of approximation for that problems where particles retain their identity.

In Sec. 2 we derive the two Heisenberg uncertainty principles in a purely geometric way in the framework of the new theory. In Sec. 3 we investigate the strong interactions and show how asymptotic freedom and confinement can be easily derived. We obtain also Regge-like relations and the renormalization constant. In sec. 4 we discuss the motion of a light ray in the Schwarzschild metric. Final considerations and directions for future work are reserved for the last Section.

2. The uncertainty principle

We consider the unidimensional motion in the Schwarzschild metric. We set A=0 and study the small oscillation of (1.7) with (1.6) and obtain

$$\left(\frac{d^2R}{dt^2}\right)-\frac{3KR}{1+KR^2}\left(\frac{dR}{dt}\right)^2+(1+KR^2)c^2KR=0. \tag{2.1}$$

In the nonrelativistic limit we arrive at the nonlinear equation

$$\left(\frac{d^2R}{dt^2}\right)+Kc^2R-3KR\left(\frac{dR}{dt}\right)^2+K^2c^2R^3=0, \tag{2.2}$$

that can be studied with a perturbative method, for example the asymptotic perturbation (AP) method [3-4]. The first order approximate solution is

$$R(t)=2\rho_0\cos(\omega t-\vartheta)_0\,, \tag{2.3}$$

where ρ_0 and ϑ_0 are fixed by the initial conditions and

$$\omega=c\sqrt{K}\,. \tag{2.4}$$

Note that the frequency is equal to linear case and is not modified by the nonlinear terms. In other words particles appear to possess a “zitterbewegung” or intrinsic trembling.

The uncertainty principle

$$\Delta X\Delta P\geq\frac{h}{4\pi}\,, \tag{2.5}$$

where ΔX and ΔP are respectively the medium square root of the spatial and momenta measurements respectively. The equation (2.5) can be easily derived if

$$\frac{1}{\sqrt{K}}=\frac{h}{4\pi mc},\ \rho_0=\frac{1}{\sqrt{2K}}\,, \tag{2.6}$$

where ρ_0 is the oscillation amplitude (2.3).

We obtain

$$E=mc^2=\frac{1}{2}(mKc^2)(4\rho_0^2)\,, \tag{2.7}$$

$$\Delta X\Delta P=(\sqrt{2}\rho_0)(m\omega\sqrt{2}\rho_0)=\frac{\hbar}{2}\,, \tag{2.8}$$

and moreover

$$\Delta E\Delta t\approx(mc^2)\left(\frac{1}{\omega}\right)\approx\frac{\hbar}{2}\,. \tag{2.9}$$

It is crucial that the frequency does not depend on the oscillation amplitude (the nonlinear correction term of the frequency depending on the square of the amplitude vanishes as we can see using for example the AP method)

In order to support the previous result we seek an approximate solution of the equation (1.2) in the linear limit. We use the diagonal metric

$$ds^2 = Adx^2 + Bdy^2 + Cdz^2 + Dc^2dt^2 . \tag{2.10}$$

We recall that the metric (2.10) is not the general case in 3+1 dimensions, but only in 2+1 dimensions.

We set

$$A \to -1 + A, B \to -1 + B, C \to -1 + C, D \to 1 + D, \tag{2.11}$$

where A, B, C, D <<1.

We seek a solution of the form

$$\alpha \exp\left\{ i\left(\vec{k} \cdot \vec{x} - \omega t \right) \right\} + P(x, y, z, t), \tag{2.12}$$

where *P(x,y,z,t)* is a second order polynomial and obtain

$$\begin{aligned} A = \alpha \exp\left\{ i\left(\vec{k} \cdot \vec{x} - \omega t \right) \right\} + a_1 x^2 + a_2 y^2 + a_3 z^2 + a_4 xy + a_5 xz + a_6 yz + \\ a_7 t^2 + a_8 xt + a_9 yt + a_{10} zt + a_{11} x + a_{12} y + a_{13} z + a_{14} t, \end{aligned} \tag{2.13}$$

$$B = A(\alpha \to \beta; a \to b), C = A(\alpha \to \gamma; a \to c), D = A(\alpha \to \delta; a \to d). \tag{2.14}$$

with

$$\begin{aligned} \omega^2 = k_1^2 , \delta = -\alpha, \beta = -\gamma, k_2 = k_3 = 0, \\ \omega^2 = k_2^2 , \delta = -\beta, \alpha = -\gamma, k_1 = k_3 = 0, \\ \omega^2 = k_3^2 , \delta = -\gamma, \alpha = -\beta, k_1 = k_2 = 0. \end{aligned} \tag{2.15}$$

The second grade polynomial is a consequence of the third derivatives of the equation (1.2).

For the validity of the approximation the terms depending on time must vanish and the other terms must form a potential hole.

Using the geodesics equation (1.7), we obtain in the 1+1 dimensional case the harmonic oscillator equation

$$\ddot{X} + c^2 d_1 X = 0, \tag{2.16}$$

in the nonrelativistic and linear limit. The equation (2.16) confirms the presence of an intrinsic trembling.

3. Confinement and asymptotic freedom for the strong interaction

Inside a hadron in order to describe strong interactions among quarks, we adopt the equation (1.7) and with θ =cost, θ = π/2 , motion along a plane, we obtain

$$R^2 \frac{d\varphi}{ds} = H, \tag{3.1}$$

$$\alpha \frac{dt}{ds} = B, \tag{3.2}$$

$$\left(\frac{d^2 R}{dt^2} \right) - \frac{3}{2\alpha} \frac{d\alpha}{dR} \left(\frac{dR}{dt} \right)^2 + c^2 \frac{\alpha}{2} \frac{d\alpha}{dR} - \frac{H^2 \alpha^3}{R^3 B^2} = 0, \tag{3.3}$$

where B and H are the motion constants.

An alternative form of the equation (3.3) is

$$\frac{d^2u}{d\varphi^2}+u+\frac{A}{2H^2}+\frac{3}{2}Au^2-\frac{K}{H^2u^3}=0, u=\frac{1}{R}. \tag{3.4}$$

If in the equation (3.4) we neglect the fourth and the fifth term, we obtain the newtonian orbits of the two-body problem. We now consider the equation (3.3) in the nonrelativistic limit (B≈1/c) and for small distortions of the metric ($\alpha \approx 1$),

$$\left(\frac{d^2R}{dt^2}\right)+Kc^2R-\frac{Ac^2}{2R^2}-\frac{c^2H^2}{R^3}=0, \tag{3.5}$$

where we find the harmonic, newtonian and centrifugal terms.

The potential for unit mass is

$$V(R)=\frac{1}{2}Kc^2R^2+\frac{Ac^2}{2R}+\frac{H^2c^2}{2R^2} \quad . \tag{3.6}$$

We study the small oscillations around a minimum, R_0, (with *A>0*, *K>0*), the equation (3.5) yields

$$\left(\frac{d^2R}{dt^2}\right)+AR+BR^2+CR^3=0, \tag{3.7}$$

$$A=\frac{3Ac^2}{2R_0^3}+\frac{4H^2c^2}{R_0^4}, B=-\frac{3Ac^2}{2R_0^4}-\frac{6H^2c^2}{R_0^5}, \tag{3.8}$$

$$C=\frac{10H^2c^2}{R_0^6}+\frac{2Ac^2}{R_0^5}, \tag{3.9}$$

The first order approximate solution is

$$R(t)=R_0+2\rho_0\cos(-\Omega t+\varphi_0), \tag{3.10}$$

with ρ_0 e φ_0 depending on the initial conditions and Ω is given by

$$\Omega=\omega\,(1+\frac{D\rho_0^2}{\omega^4}), \tag{3.11}$$

$$\omega=\sqrt{A}\,, \qquad D=-\frac{3A^2c^4}{4R_0^8}-\frac{9AH^2c^4}{2R_0^9}. \tag{3.12}$$

The weak nonlinearities in (3.17) cause a frequency renormalization from ω (3.12) to Ω (3.11). The harmonic motion (3.10) is very important because it can explain well-known results about hadronic mass spectra obtained just postulating the existence of the harmonic motion. We consider now the unidimensional motion and from the equation (3.3) with *H=0* we obtain

$$\left(\frac{d^2R}{dt^2}\right)-\frac{3}{2\alpha}\frac{d\alpha}{dR}\left(\frac{dR}{dt}\right)^2+c^2\frac{\alpha}{2}\frac{d\alpha}{dR}=0, \tag{3.13}$$

For the small oscillations around the equilibrium point

$$R_0 = \sqrt[3]{\frac{A}{2K}}, A > 0, K > 0 \quad \text{or} \quad A < 0, K < 0 \tag{3.14}$$

we obtain

$$\left(\frac{d^2R}{dt^2}\right) + 3Kc^2\alpha_0 R + \frac{3Kc^2\alpha_0 R^2}{R_0} - \frac{9KR}{\alpha_0}\left(\frac{dR}{dt}\right)^2 - \frac{Kc^2(3+\alpha_0)R^3}{R_0^2} = 0, \tag{3.15}$$

$$\alpha_0 = 1 + 3KR_0^2\ . \tag{3.16}$$

The first order approximate solution is

$$R(t) = R_0 + 2\rho_0 \cos(-\Omega t + \varphi_0) \tag{3.17}$$

$$\Omega = \omega\left(1 - \frac{11\rho_0^2}{3R_0^2}\right), \tag{3.18}$$

$$\omega = c\sqrt{3K(1+3KR_0^2)}\ , \tag{3.19}$$

where $\rho_0,\ \varphi_0$ are given by the initial conditions.

Also in this case, the weak nonlinearities in (3.15) cause a frequency renormalization from ω (3.19) to Ω (3.18).

If we consider the Newtonian limit (v<<c), we obtain

$$\left(\frac{d^2R}{dt^2}\right) + \frac{dV(R)}{dR} = 0, \tag{3.20}$$

where

$$V(R) = \frac{c^2}{4}\left(1 + \frac{A}{R} + KR^2\right)^2. \tag{3.21}$$

For the small oscillations around a stable equilibrium position R_S, we obtain the equation (3.10), but with

$$\Omega = \omega\left(1 + \frac{D\rho_0^2}{\omega^4}\right), \tag{3.22}$$

$$\omega = \sqrt{A}, A = c^2(3K + 9K^2R_0^2\ ), \tag{3.23}$$

$$D = \frac{117K^3}{2} - \frac{3K^2}{R_0^2} + \frac{297K^4R_0^2}{2}. \tag{3.24}$$

We now show how the Schwarzschild metric can be used to study the interior of an hadron and consider the potential (3.21), that presents both the confinement and the asymptotic freedom. Our analysis is rigorously applicable only to a tiny mass *m* that moves in the field of a large mass M.

If we want to include the centrifugal effects we must use the equation (3.3) that for intermediate distances ($H\alpha \approx H$) is equivalent to the potential

$$V(R) = \frac{c^2}{4}\left(1 + \frac{A}{R} + KR^2\right)^2 + \frac{J^2}{2m^2R^2}, \tag{3.25}$$

with the angular momentum J,

$$J = mcH, \tag{3.26}$$

or more simply

$$V(R) = \frac{Ac^2}{2R} + \frac{J^2}{2m^2R^2}\,. \tag{3.27}$$

In the newtonian limit there are circular orbits where the force vanishes with the radius

$$R_0 = -\frac{J^2}{Am^2c^2} \tag{3.28}$$

and where we observe the asymptotic freedom. Near the value (3.28) the hadron constituents behave as if they were almost free particles. Note that in this case the expression (3.28) is valid even when the particle of mass m does not possess a small mass with respect to the mass M. In order to describe strong interactions inside a meson, we set the following values for the constants

$$A = -\frac{2SM}{c^2}, \qquad S = \frac{\hbar c}{m^2}, \tag{3.29a}$$

We know indeed that for the quark-quark-gluon coupling

$$\frac{Sm^2}{\hbar c} = 0.2, \tag{3.29b}$$

bur however the value will vary with the particular hadron chosen for the comparison. From the equation (3.28) we obtain a typical value for the meson radius

$$R_0 = \frac{J^2}{Sm^3} \approx \frac{\hbar}{mc} \approx \ 1\,fm, \tag{3.30}$$

with

$$m \approx M \approx \frac{1}{3}(\text{proton mass}), \tag{3.31}$$

From (3.28) we obtain also

$$\frac{J}{\hbar} \propto m^{\frac{3}{2}} \approx m^2, \tag{3.32}$$

i.e. the well known Regge relation.

A more rigorous result can be obtained using the equation (6.9).

Typical values for the constants are

$$S = 0.2(Gev^{-1}c^4\,fm), A = -0.4Mc^2(Gev^{-1}\,fm), \tag{3.33a}$$

$$K = 6(Mc^2)^{-2}(Gev^2\,fm^{-2}), \tag{3.33b}$$

At very large distances, when R is greater of the hadron radius (3.30), we obtain from (3.25) an attractive radial force

$$F = -mc^2KR(1 - \frac{2SM}{c^2R} + KR^2)\,, \tag{3.34}$$

i.e. a confining force able to explain the confinement of quarks. Note that the confinement can be obtained also for negative values of K (for large R the K^2 term is dominant).

It is well known that in the case of a spherically symmetric static and diagonal metric, the Lorentz factor is proportional to $\sqrt{g_{00}}$. The strong coupling constant is then

$$\alpha_S(R) = \frac{SM^2}{\hbar c\left[1 - \dfrac{2SM}{Rc^2} + KR^2\right]}, \tag{3.35}$$

and taking into account the relativistic mass variation we newly derive both the asymptotic freedom and confinement. Note that the strong coupling constant (3.35) is analogous to the perturbative coupling constant of the standard theory and contains both asymptotic freedom and confinement.

The energy of a particle of mass m in the stable circular orbit is

$$E = mc^2\sqrt{\frac{R_0}{R_0 + A + KR_0^3}}, \tag{3.36}$$

where R_0 is the equilibrium point. Using the equation (3.28)., we obtain with the second of (3.29a) the approximate relation

$$J = \frac{\hbar\sqrt{2}}{\sqrt{\dfrac{E}{mc^2} - 1}}. \tag{3.37}$$

4. The motion of a light ray into a hadron

For the motion of a light the condition is

$$ds^2 = 0, \tag{4.1}$$

that for radial motion yields from (1.5-1.6)

$$\frac{1}{2}\left(\frac{dR}{dt}\right)^2 + V(R) = 0, \tag{4.2}$$

i.e. the classic motion of a body with zero energy in the potential

$$V(R) = -\frac{c^2}{2}\left(1 + \frac{A}{R} + KR^2\right)^2, \tag{4.3}$$

We seek the positive roots of the cubic equation

$$R^3 + \frac{R}{K} + \frac{A}{K} = 0, \tag{4.4}$$

and find three cases: i) only one real root

$$R_1 = u + v, \tag{4.5}$$

$$u = \sqrt[3]{-\frac{A}{2K} + \sqrt{\frac{A^2}{4K^2} + \frac{1}{27K^3}}}, \tag{4.6}$$

$$v = \sqrt[3]{-\frac{A}{2K} - \sqrt{\frac{A^2}{4K^2} + \frac{1}{27K^3}}}, \quad (4.7)$$

if

$$A^2 > -\frac{4}{27K}, \quad (4.8)$$

ii) two real roots

$$R_1 = u + v, R_2 = R_3 = -\frac{u+v}{2}, \quad (4.9)$$

if

$$A^2 = -\frac{4}{27K}, \quad (4.10)$$

c) three real roots

$$R_1 = 2\sqrt[3]{r}\cos\frac{\varphi}{3}, R_2 = 2\sqrt[3]{r}\cos(\frac{\varphi}{3} + 120^0), \quad (4.11)$$

$$R_3 = 2\sqrt[3]{r}\cos(\frac{\varphi}{3} + 240^0), r = \sqrt{-\frac{1}{27K^3}}, \quad (4.12)$$

$$\cos\varphi = -\frac{A\sqrt{-27K^3}}{2K}, \quad (4.13)$$

If

$$A^2 < -\frac{4}{27K} \quad (4.14)$$

These roots are acceptable only if are positive. In these cases light does not enter into the hadron and we can get a valid model of a static particle (micro black hole).

5. Conclusion

We have introduced a purely geometric nonlinear and non-local modification of the General Relativity and we have derived the most important properties of strong interactions, confinement and asymptotic. The uncertainty principle emerges naturally in this model as consequence of the non-local modification of the General Relativity. At the same time, the dichotomy matter-field of the Einstein equation is eliminated and the physical world is described only by means of a unified field.

Moreover, we note that in the last years a purely classical approach by Einstein-type equations (with the cosmological term Λ) has been accomplished to explain strong interactions and hadron structure in the usual 3+1 spacetime dimensions [42-44], without introducing extra dimensions. Along these guidelines, an unified bi-scale theory of gravitational and strong interactions can be constructed and, in particular, hadrons are identified with ‘strong’ black-holes, i.e. suitable stationary and asymmetric Kerr-Newman-de Sitter solutions of this Einstein-type equations. As a consequence, the constant Λ and the masses result to be scaled up and transformed into a hadronic constant and strong masses respectively. Using this approach, confinement and asymptotic freedom have been easily derived. This theory is different from that exposed in the present paper in the fundamental postulates but similar in the physical results about strong interactions.

From the above considerations, it seems possible the interpretation of strong interactions by means of the new equation (1.2). Future papers we will investigate in detail the identification of elementary particles with lumps of the field, i.e. zones where the field assumes particularly high or infinite values (black holes).

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