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# Determining fermion masses and resolving the problem of generations: A clue to incorporate Gravity into the Standard Model

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## Abstract

Inspired by the Poincare model of the electron, elementary fermions are assumed to be bubble like structures with negative internal pressure and a size corresponding to a radius or equivalently an ultraviolet cut-off. Negative pressure, self-interaction of gauge fields up to the cut-off energy and gravity contribute to the self-energy. All corrections are considered to be proportional to the observed mass of the fermion in order to preserve chiral symmetry in the limit of vanishing fermion mass. Fermion self-energy is thus constituted of terms; inverse cubic, logarithmic (as in qed self-energy of the electron), linear and quadratic in the cut-off parameter and defines coupling coefficients. The latter two terms originating from gravity are fixed on basis of dimensional considerations. The condition of extremization of total energy to determine equilibrium states, leads to a quantic equations with three real roots, giving three values for the fermion mass, for each set of coupling coefficients. The model represents observed quark – lepton mass ratios explaining generation problem and suggest possible numerical values for neutrino masses in agreement with the oscillation data in an inverted order. As a result of incorporation of gravity, Planck energy sets as the natural physically meaningful scale, deriving other scales corresponding to sizes of elementary fermions ranging from Planck length to few thousand times this unit. The model interprets, the physical quality distinguishing a generation as the phase of the false vacuum 'inside' the elementary bubble. The unconventional approach behind the model may also have implications on unifications of couplings, incorporation of gravity into the standard model and issue of divergences in quantum field theories.

***Keywords*:** Fermion generations, Lepton-quark masses, Neutrino masses, Standard Model and gravity, Beyond Standard Model

## 1. Introduction

The standard of model (SM) is tremendously successful in explaining pattern of the hadron spectrum and calculating scattering amplitudes involved in interactions between leptons and quarks mediated by gauge bosons [1-2]. Spontaneous breaking of SU (2) x U (1) symmetry in SM accounts for existence massive vector bosons, leptons and the scalar Higgs particle [1-6]. However, SM fails to determine the masses of leptons and quarks and explain why they replicate into three generations

[7-11]. Again SM demands neutrinos to be massless, contrary to the evidence from neutrino oscillation experiments, which clearly indicate that these particles possess definite but minute mass [12-13]. Presence of widely separated energy (mass) scales and leaving gravity aside are other undesirable features of the SM. The critical of problem of SM is its inability to compute the self-energies of the basic entities, as expressions derived turns out be divergent in the limit of point particles. The renormalizability [14] of the theory, enables calculations of many useful observables except masses.

In classical models of the electron, the divergence of the self-energy was avoided by assigning a finite size to the election. Abraham and Lorentz considered electron as bubble of radius $r$ with an evenly distributed surface charge $Q$. Equating electrostatic energy $Q^2/(4\pi\varepsilon_o r)$ to $m_e c^2$, a finite value is obtained for electron radius [15-16]. Poincare noted that the above model is unstable and introduced a non-electromagnetic stress to stabilize the electron [17]. In modern language Poincare stress is equivalent to a negative pressure $P$ inside the bubble. Thus mass $m$ of the bubble can be written as,

$$m = (4\pi r^3 P)/3c^2 + Q^2/(4c^2\pi\varepsilon_o r) \tag{1}$$

The expression (1) is minimum when $r = r_e = Q^2/(32\pi\varepsilon_o P)$ , giving

$$m_{min} = Q^2/(6\, c^2\pi\varepsilon_o r_e) = m_e \tag{2}$$

The electron radius $r_e$ obtained from (2) is nearly three orders of magnitude smaller than the electron Compton wavelength – a value ruled out by experiment and also untenable, because at this dimensions of length, QED vacuum polarization overrules classical electrostatics. In a recent note, the author examined a semiclassical model of leptons [18], where the second term in (1) is replaced by the QED expression for correction to electron self-energy [19] given by,

$$(\delta m)_{\text{qed}} = (3\alpha/2\pi)\, m\, [\ln\{\hbar/(mcr)\} + 1/4] \tag{3}$$

so that,

$$m = (4\pi r^3 P)/3c^2 + (3\alpha/2\pi)\, m\, [\ln\{\hbar/(mcr)\} + 1/4] \tag{4}$$

The expression (4) is minimum when,

$$r = (\hbar/(mc)) \exp\{-(2\pi/3\alpha) + 7/12\} \tag{5}$$

At the lengths scales involved in (5), the fine structure constant $\alpha$ could be renormalized to a value nearly an order of magnitude larger than infrared limit (~ 1/137) and if the tau lepton radius is assumed to be of the order of Planck length, radii of the muon and electron turns out one and two orders of magnitude larger [18].

Inclusion of gravity to generate finite self-energies in classical and quantum objects has been an attractive hypothesis receiving continued attention [20-29] .Here I extend the idea discussed above by incorporating contributions to self- energy of elementary fermions expected to originate from gravity. The results are encouraging and seem to provide clues to resolve puzzles of lepton-quark generations their mass hierarchies and the problem of divergences in quantum field theories.

## 2. Discussion

An important point that needs be emphasized is the fact the QED correction to electron self-energy $(\delta m)_{\text{qed}}$ given by (3) is proportional to observed electron mass [2, 30]. Essentially, this is a requirement that in limit of zero fermion mass, the chiral symmetry of the theory is guaranteed - a condition originating from the fact that the term $m\psi\,\psi$ in the Lagrangian is not invariant under the transformation $\psi \rightarrow exp\ i\theta\gamma_5\psi$. Similarly to preserve the chiral symmetry in the limit of zero mass, all the other correction to the self-energy of elementary fermions should be proportional their masses. Again higher powers of $m$ in corrections should also be excluded because the mass term in the corrected the final Lagrangian should also look like $m\ \bar{\psi}\,\psi$ once all corrections are included. Therefore I conjecture that $P$ in (4) denoting negative pressure should be written as,

$$P = m\kappa \tag{6}$$

where $\kappa$ = a constant so that the correction to the self-energy originating from negative pressure $(\delta m)_{\text{np}}$ [first term in Eqn. (4)] is,

$$(\delta m)_{\text{np}} \quad = \quad (4\pi r^3 m\ \kappa)/3c^2 \tag{7}$$

Now I look for corrections to self-energy originating from gravity that are directly proportional to mass. In absence of an adoptable quantum theory of gravity, I simply consider dimensions and arrive at two terms $-(GmM_p)/c^2r$ and $Gme^2/\ (c^2\varepsilon_o r^2\ )$ , where $G$ = gravitational constant, $M_p$ = Planck mass and $e$ = electronic charge. The latter term is taken to be positive for the following reason. Suppose the bubble contracts due its own gravitation, then the electric field (vacuum polarization) increases – giving gravity coupled to electromagnetism a positive contribution. The other possible terms will be of higher order in $G$ or $e^2$. Thus the contribution to self-energy from gravity $(\delta m)_{\text{gr}}$ can be written as,

$$(\delta m)_{\text{gr}} = -\ (\eta GmM_p)/c^2 r \quad + (\gamma Gme^{2)}/\ (c^2\varepsilon_o\, r^{\,2}) \tag{8}$$

where $\eta$ and $\gamma$ are dimensionless constants of proportionality.

Hereinafter I proceed with units $\hbar$ = c = $G$ = 1 so that Planck length $L$p = 1 and Planck mass $M_p$ = $1$ and replace $r$ by $\Lambda^{-1}$ , to transfer from length scale $r$ to an energy scale $\Lambda$, measured in Planck units. With this simplification, self-energy- the sum of corrections [ $(\delta m)_{\text{np}} + (\delta m)_{\text{qed}} + (\delta m)_{\text{gr}}$] takes the form,

$$m = [(4\pi m\kappa)/3]\ \Lambda^{-3} \ + \ 3(\alpha\, m)/2\pi[\ln{(\Lambda/m)} + 1/4\}] \ - \eta m\Lambda \ + \gamma\alpha m\Lambda^2 \tag{9}$$

The last term in (9) is the much feared quadratic divergence which workers strive to eliminate. Contrary to this belief, it is an important attribute that could save SM of its flaws. The number $\alpha$ in (9) should now be considered as a unified gauge coupling constant the yielding a logarithmic divergence for leptons as well as quarks. The common multiplicative $m$ ($\neq$ 0) in (9) disappears and (9) can also written as,

$$\begin{aligned} m \ = \ & \Lambda\, exp\{\ [(8\pi^2\ \kappa))/9\alpha]\Lambda^{-3}\ -\ [\ ([(2\pi\eta)/3\alpha]\ \Lambda \ + \ [(2\pi\gamma)/3]\ \Lambda^2 \ - [2\pi/3\alpha - \tfrac{1}{4}]\ \} \\ = \ & \Lambda\, exp\ \{\ (d/3b)\Lambda^{-3}\ -\ (a/b)\ \Lambda \ + \ (1/2b)\ \Lambda^2 \ - (2\pi/3\alpha - \tfrac{1}{4})\ \} \end{aligned} \tag{10}$$

where the positive constants $a$, $b$ and $d$ are related to coupling coefficients $\eta$, $\gamma, \alpha$ via relations,

$$a = \eta/(2\gamma\alpha) \ , \ b = 3/(4\pi\gamma) \ , \ d = (2\pi\kappa)/\gamma\alpha \tag{11}$$

The condition $dm/d\Lambda = 0$ yield the quantic equation,

$$\Lambda^5 - a\Lambda^4 + b\Lambda^3 - d = 0 \tag{12}$$

Again from (10), at $dm/d\Lambda = 0$,

$$d^2m/d\Lambda^2 = m\,(b\Lambda)^{-1}\,[3d\Lambda^{-4} - a + 2\Lambda] \tag{13}$$

Thus real roots $\Lambda_i$ of the quantic (12) corresponds to equilibrium values of $m$ and using (10), (11) and (12), the allowed masses $m_i$ can be expressed as,

$$m_i = C_o\Lambda_i\, exp\,[(5/6b)\Lambda_i^2 - (4a/3b)\,\Lambda_i\,] \tag{14}$$

where,

$$C_o = exp - [2\pi/3\alpha - 7/12] \tag{15}$$

Note that the constant $d$ does not appear explicitly in (14), masses $m_i$ depend on constants $a$, $b$ and $\Lambda_i$ derived from (13) . Suppose the variable $\Lambda$ in (12) is transformed to $\Lambda' = f\Lambda$ , where $f$ is a scaling factor the coefficients of the quantic will transform as $a' \rightarrow af$ , $b' \rightarrow bf^2$ , $d' \rightarrow df^5$. Under the same transformation (14) will transform as,

$$m'_i = [\,(C_o f\,)\Lambda_i\,]\, exp[\,(5/6b')\,\Lambda_i^2 - (4a'/3b')\,\Lambda_i\,] \tag{16}$$

Thus the mass ratios $m_{i/}\ m_i$ are unaltered by the scale transformation. As detail information regarding the values of coefficients $a$, $b$ and $d$ defined by (11) are unavailable, the above property of (12) opens an avenue to compare the mass ratios by fixing one of the coefficients ($a$ or $b$) arbitrarily with re-definition of the constant $C$ in (14) as $Cf$. Therefore without loss of generality, I set $b = 1$ in (11), (12) and (14) and confine subsequent analysis to the following simplified equations,

$$\Lambda^5 - a\,\Lambda^4 + \Lambda^3 - d = 0 \tag{17}$$

$$m_i = C\Lambda_i\, exp\,[(5/6)\,\Lambda_i^2 - (4a/3)\,\Lambda_i\,] \tag{18}$$

where $C = C_o f = f\, exp - [2\pi/3\alpha - 7/12]$

Equation (17) has three real positive roots provided,

$$a \geq 2 \quad \text{and} \quad d \leq d_L \tag{19}$$

where $d_L$ is a limiting value dependent on $a$, but not expressible as a familiar function of $a$ (well-known algebraic property of quantic equations [31] ). Thus it follows from (18), when (19) is satisfied there are three equilibrium points of (10) corresponding three masses $m_1$, $m_2$, $m_3$ . Fixing values of $a$ and $d$ defines a generation of three members.

It is easy to show that the possible values of $m_i$ for each generation has a maximum $m_{max}$ and minimum $m_{min}$ and their ratio is given by,

$$m_{max}/m_{min} = [\,1 + \sqrt{(1 - 15/4a^2)}]\,[1 - \sqrt{(1 - 15/4a^2)}]^{-1}\, exp\,\{\,-4a/5\sqrt{(1 - 15/4a^2)}\,\} \tag{20}$$

As $a \geq 2$, it follows from (20), that the three masses in each generation has lower and upper bounds $m_L$ and $m_U$ and their ratio satisfy the relation,

$$m_L/m_U > (5/3)\, exp\,(-2/5) = 0.11176 \tag{21}$$

The above result derived independently of the value of any constants is obviously satisfied by leptons and quarks and tells that elementary fermions needs to be massive. Masses of these fermions, three in number are bounded from below and above.

Except in special cases, the roots of a quantic equation cannot be expressed as a formula involving the coefficients of the powers of the variable $x$ [31]. Numerical analysis reveal that that equations (17) and (18) have solutions agreeing with the observed mass ratios of charged leptons and quarks. When one mass ratio is used as an input to fix the unknown parameters, the other ratio computed, agrees with observation.

***Case I*** $a = 4.9599$, $d = 3.700 \times 10^{-8}$. Here roots of the equation (17) are $\Lambda_1 = 4.74934$, $\Lambda_2 = 0.21055$, $\Lambda_3 = 3.351 \times 10^{-3}$. Inserting these values in (18), the ratio of masses obtained is $m_1 : m_3 : m_2 = 1 : 206.7 : 3429$. The observed charged ($Q = 1$) lepton mass ratio is $m_e : m_{mu} : m_{tau} = 1 : 206.8 : 3482$

***Case II*** $a = 5.5926$ , $d = 3.508 \times 10^{--11}$. Corresponding roots of the equation (17) are $\Lambda_1 = 5.40768$, $\Lambda_2 = 0.18492$, $\Lambda_3 = 3.280 \times 10^{-4}$. Insertion of these values in (18), gives the mass ratio $m_1 : m_3 : m_2 = 1 : 514 : 75262$. The observed Q = 2/3 quark mass ratio is $m_{up} : m_c : m_t = 1 : 554 : 75309$.

***Case III*** $a = 4.646$ , $d = 2.30 \times 10^{-9}$, yield roots $\Lambda_1 = 4.4197$, $\Lambda_2 = 0.2263$, $\Lambda_3 = 1.324 \times 10^{-3}$. Inserting these values in (18), the ratio of masses obtained is $m_1 : m_3 : m_2 = 1 : 19.7 : 871$. The observed $Q = 1/3$ quark mass ratio is 1: 19.8: 872.

It is interesting to note that the increasing order of mass values ($m_1 : m_3 : m_2$) is not the decreasing or the roots ($\Lambda_1 : \Lambda_2 : \Lambda_3$). The result is understandable because in the expression (18) for $m_i$ depends on $\Lambda_i$ linearly as well as exponentially and exponential factor is negative when $\Lambda < 5a/5$. Furthermore, it follows from (13) that $m_1$ and $m_3$ corresponds to minima of $m$, whereas $m_2$ happens to be a maximum. In this situation, tau lepton and bottom and top quarks are unstable equilibrium points of $m$ and all the other quarks and charged leptons corresponds to minima of $m$ (Fig.1).

Masses $m_1$ corresponding to electron, up and down quarks is an absolute minimum implying there stability (this does not forbid up, down quark transformation via virtual W boson exchanges in energetically permitted interactions). Muon, charm and strange quarks corresponds to metastable position $m_3$ and $m_2$ represents unstable points of equilibrium corresponding to tau lepton and top and bottom quarks

It is interesting that there is indeed a reason why the tau lepton, top quark or the bottom quark sitting on peak 2 (Fig.1) are not falling down to the valleys on either side 'instantaneously'. Flavor changing neutral currents are forbidden by SM and seem to be heavily suppressed according to measurements [32]. .Even classically, an equilibrium corresponding to a maximum in energy is not always ruled out of existence. A boulder in a valley between two mountains is absolutely stable. A boulder can also safely sit on the mountain peak, but never on a slope. Small extraneous influences can stabilize systems in equilibrium with maximum energy. Well studied example is the classical and quantum inverted pendulum [33]. When it comes to rapidly decaying particles it is hard to distinguish metastability and unstable equilibrium.

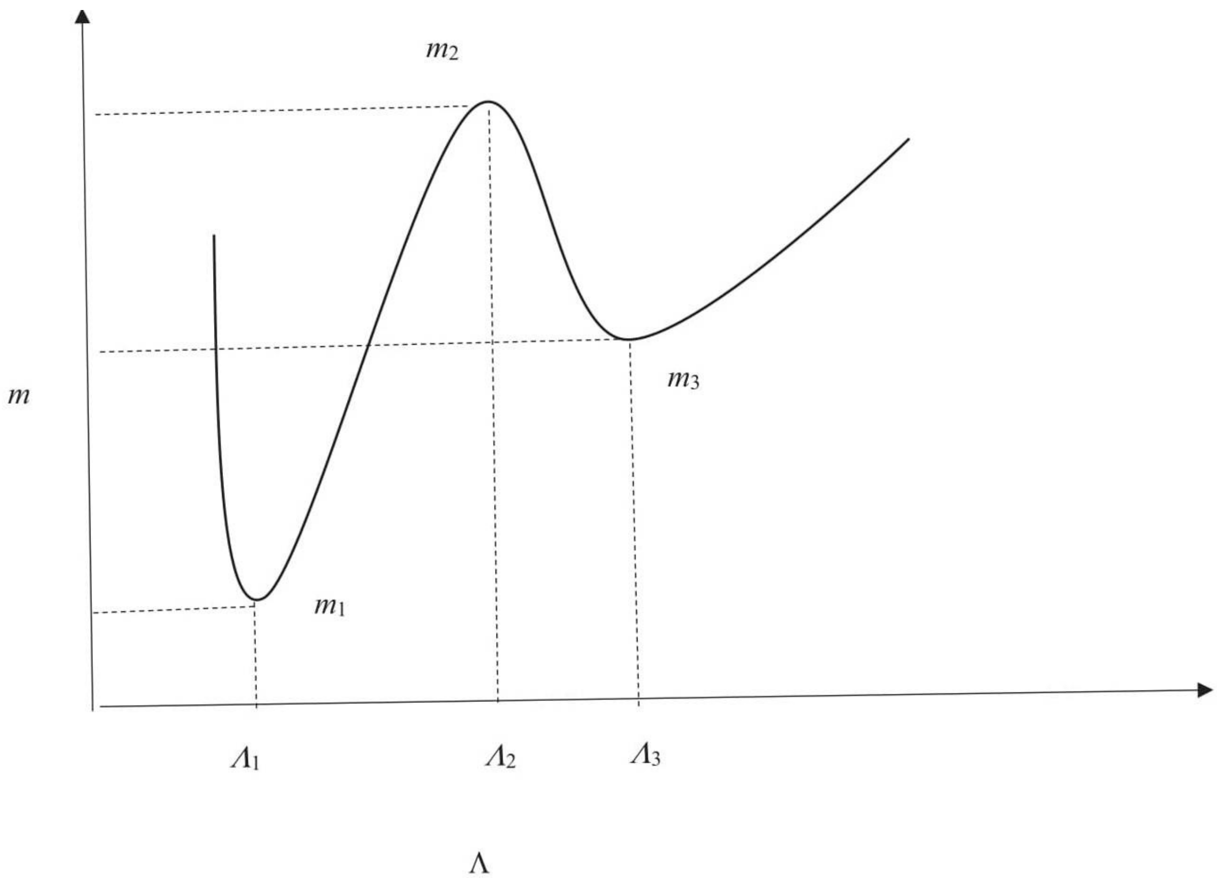


Fig.1. Sketch showing minimum $m_1$ representing masses of electron, up quark or down quark; a maximum $m_2$ corresponding to masses of tau lepton, top quark or bottom quark and second minimum representing masses of muon, strange quark or charm quark taken that order ( figure not drawn to represent the scales involved)

*Case IV* $a = 2$. This is the smallest possible value of $a$ giving smallest possible masses according to the model and could correspond to neutrinos. Neutrino oscillation data indicate that the character of their mass ordering is two particles of nearly equal mass and a third with a significantly smaller or larger value. When $a = 2$ and $d = 0$, equation (17) has two equal roots each equal to unity and three zero roots, leading to two degenerate non - zero masses via (18). If $d$ is non-zero and small ($d < d_L = 0.03455$), the two roots manifest a small difference and a third acquire a small positive value and remaining two roots become complex. Thus according to (18) there will be two nearly equal masses and smaller third mass. Such solutions of (17) and (18) exists when $d$ is of the order of $10^{-2}$. The mass ratios obtained by setting $a = 2$, $d = 0.0142$ agrees with neutrino oscillation data. The roots of (17) obtained here are $\Lambda_1 = 1.10288$, $\Lambda_2 = 0.84718$ , $\Lambda_3 = 0.31020$ and the mass ratio calculated using (18) is $m_3 : m_1 : m_2 = 1 : 1.089 : 1.092$. Thus neutrino oscillation experimental value $\Delta_{23} = m_2^2 - m_3^2 = 2.50 \times 10^{-3}$ eV$^2$, leads to masses $m_3 = 11.42 \times 10^{-2}$, $m_1 = 12.47 \times 10^{-2}$, $m_2 = 12.50 \times 10^{-2}$ eV giving $\Delta_{21} = m_2^2 - m_1^2 = 7.5 \times 10^{-5}$ eV$^2$. Here again this is not mere fitting data. The input $\Delta_{23}$ yields neutrino masses and $\Delta_{21}$ derived agrees with the experimental limit. The result hints to the idea that all elementary fermion masses could originate from the same dynamical mechanism. Here again, $m_3$ and $m_1$ are minima and $m_2$ a maximum. It is interesting that model explain the hierarchical pattern of charged lepton and quark masses and suggest a quasi-degenerate inverted ordering for neutrino masses [34-35]. Above neutrino mass values and ordering are not ruled out by latest analysis of experimental data [42].

It appears that according to the model, masses of all observed elementary fermions are represented in the equation (17).

Inserting the observed values of masses for the sectors Q = 1, 1/3, 2/3 and values of neutrino masses calculated on basis of the model using neutrino oscillation data for Q = 0 in Eqn. (18) ,the quantity $[1/\alpha - (3/2\pi)\ln f]$ can be evaluated . The numbers obtained are ~ 19.6, 17.4, 19.9 and 30.0 for Q = 1, 2/3, 1/3 0 respectively The model requires that $\alpha$ is a constant for a sector of given Q, The similarity of above numbers for all charged fermions , suggest that the gauge coupling constant $\alpha$ could also be universal (same for all quarks as well as charged leptons) and the factor $f$ varies slightly from sector to sector ( because of the variation of other constants ) .

The model explain occurrence of three generations of fundamental fermions, consistent with their observed masses. Mass generation dynamics depends on three coupling constants: (i) gravitational constant $G$ (ii) a gauge coupling constant $\alpha$ (iii) a constant $\kappa$ relating fermion masses to a an energy density $P$ via the relation $P = m\ \kappa$ . Constant G is universal and the results suggest that $\alpha$ may also be universal (same all charged fermions and the value of $\alpha$ corresponding neutrinos (1/30) is almost exactly equal to so-called weak fine structure constant $\alpha_W$ which compares strengths of weak and electromagnetic interactions.

In the present model, dynamics create masses of elementary fermions. It is well known that fermion masses could be generated without invoking Higgs mechanism and many such models formulated does not contradict SM [39]. In this scenario, Higgs mechanism give masses to vector bosons via spontaneous symmetry breaking and the necessary non-zero value for its vacuum expectation value demand existence of massive fermions. This argument is fully consistent with SM and all experimental data, including observation of the Higgs boson. Yukawa coupling constants are non-zero proportionality parameters. According to the present model occurrence of three lepton – quark generations, and observed non-zero masses demand $\kappa \neq 0$ as $P = m\ \kappa$ (Eqn.8). From condition $b = 1$ and (11), it follows that $\kappa = d\alpha/2\pi$. Thus, all elementary fermions with given value of $Q$ corresponds to same value of $\kappa$. If $P$ which is a negative pressure or positive energy density is written as $M_v^4$, the values of this mass scale $M_v$ calculated from the model (using values of $d$) are approximately 6 x$10^9$, 8 x $10^{10}$ and 6 x $10^9$ GeV (Planck mass taken as 1.2 x$10^{19}$ GeV) respectively for first, second and third generations. Thus each generation could be considered as corresponding to three phases of a nearly degenerate false vacuum and flavor an attribute associated with the phase. In the picture analogous to the Poincare model of the election, the phase of the vacuum inside the bubble exits in three phases. The interesting possibility of existence of vacuum levels above the Higgs minimum has been considered previously [36-38]. Other scales appearing the model are values of $\Lambda_i$ ranging from the order of Planck scale to around $10^{-3}$ times less. Highest scale encountered in the model corresponding to the largest value of $\Lambda_i$ is 6.5x $10^{19}$GeV- possibly an ultimate cut-off or equivalently a size in the Planck scale. According to the model smallest elementary entity is the up-quark (subject to somewhat ill-defined mass of up-quark used in the calculation) followed by the electron and the largest is the top quark.

The model suggests that symmetry breaking scale responsible for fermion mass generation should be of the order of $10^{10}$ GeV (lowest of the scales involved ) or larger and the therefore does not contradict experimental results. In this scenario , SM electroweak symmetry breaking via Higgs mechanism give masses to vector bosons and Higgs field couples to massive leptons.

The proportionality of the term $(\delta m)_{gr}$ [eqn.(8) ] to $G$ and $m$ neatly accommodate Planck energy scale.. The coefficients $\gamma$ and $\eta$ may be calculable when the model is developed into a theory combining gravity and SM. The highest energy scale in the model is of the order of the Planck scale ($M_P = \sqrt{[\hbar c/G]} = 1$).The other energy scales – the values of $\Lambda_i$ (ranging from 5 – $10^{-4}$ of the Planck scale) are dynamically generated by the model and not constrained by experiment. Again these values looked upon as ultra-high energy cut-offs will not contradict SM renormalization scheme and in a way resolve the divergence problem. These scales which appear as divergence in conventional QFT thinking are points of stabilization of the self-energy.

## 3. Conclusion

The basic idea of the model is minimal finite size of elementary fermions as a 'bubbles' and forces above its radius, described by gauge fields contributing to mass. To achieve stability as in the Poincare classical model, the bubble inside is assumed to possess a negative pressure. This pressure also contribute to mass. The other known force gravity also contribute to mass and possible corrections identified on basis of dimensional considerations. A crucial argument abided by has been assuming direct proportionality all corrections to observed mass itself, so that preservation of chiral symmetry guaranteed in the massless limit. Total energy (mass = $m$) of the bubble originating from all contributions is a function of the radius $r$ or equivalently a cut-off energy scale $\Lambda$. The condition of equilibrium imposed as an extremization of energy leads to a quantic equation with three real roots corresponding three masses. The values of the coefficients of the equation (17) remain undetermined and quantic equations are insolvable in terms of familiar functions. Consequently, the numerical analysis required to determine if the solutions, could represent observed fermion masses was a formidable problem. Fortunately, the observation that mass ratios given by the model remain scale invariant enabled simplification and use a quantic equation solver [42] to perform most calculations. The solutions represent the mass ratios of leptons and quarks remarkably close to the observed values, explaining mass hierarchy. It is very unlikely that a conventional perturbative approach will be able to explain occurrence of three quark lepton generations with masses that appear to scale unnaturally. Model has also made suggestions regarding neutrino masses. The values of masses of obtained are consistent with neutrino oscillation data and other limits.

The model suggest that the core of elementary fermions belonging to a generation is associated with distinct phase of false vacuum so that there are three phases corresponding to flavor. The fundamental nature of phases of matter has been recognized by ancients as well as modern physicists. Gautama Buddha preached that universe is constituted four basic entities; 'patavi' (earth-solid), apo (water-liquid), 'vayo' (air -gas), 'thejo' (fire-plasma). The first attempt to understand phase transitions by Van der Waals led to a cubic equation. A recent model attempting to explain the three phases of matter (solid, liquid, gas) is based on a quantic equation [41], very similar to the one presented in this work. It is amazing to speculate why false vacua also exists in three phases.

The last challenge of classical electromagnetism was explaining mass and inertia as electromagnetic consequences. When the idea failed Poincare introduced non-electromagnetic forces. The next attempt was QED calculation of the electron self-energy, which turned cut-off dependent and logarithmically divergent. However, the self-energy problem in QED paved way for the renormalization scheme. The standard model strongly suggest that, the problem of mass reside largely in the electroweak sector. However, explaining lepton-quark masses doesn't seems within its domain indicating the necessity of invoking other forces and the only known force not connected to SM is gravity. Color forces probably have little significance in determining fermion mass spectrum, especially if the masses generated at a unification scale. Thus a reasonable, first step in going beyond SM should be amalgamation of SU(2) x U (1) and gravity. The present model suggests that such a scheme will enlighten, understanding of the problem of mass, resolving flaws in SM. Point mass seems to be an impossibility and minimum length scales have been considered previously [44]. With the length scales involved here, there is no practical violation of Lorentz invariance. Obtaining full consistency for minute finite sizes would be a matter for a future theory.

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