Science Front Publishers *Journal for Foundations and Applications of Physics*, **vol. 5, No. 2 (2018)**
(sciencefront.org) **ISSN 2394-3688**

A Semiclassical Model of Leptons

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(Received 15 September 2018, Accepted 01 October 2018, Published 15 October 2018)

Abstract

A semi-classical model of leptons is presented on the assumption that they are stable equilibrium states of spherical bubble like extended structures with negative pressure of a false vacuum created inside and balanced by an outward stress due to vacuum polarization originating from the charge residing on the surface. The idea is a semiclassical analog of the Poincare model of the electron, where the outward classical electromagnetic stress is replaced by the stress due to vacuum polarization. Here the electron carries a bare mass (energy) due to negative pressure or equivalently a positive energy density inside and QED electromagnetic self-energy and both dependent on a cut-off radius R. Minimization of total energy with respect to R, yields a relation connecting equilibrium radius, negative pressure P, renormalized fine structure constant and lepton mass. Assumption that the maximum possible value of P corresponds most massive tau lepton is Planck pressure, enables determination of the renormalized fine structure constant and input of masses of the electron and muon determines corresponding internal negatives pressures and lepton radii. Tau lepton size is of the order of the Planck length and the muon and the electron are two and three orders of magnitude larger. Model suggests that the lepton flavor is an attribute associated with three different phases of a false vacuum.

***Keywords*:** Leptons, Electron self-energy, Lepton masses, Lepton flavor

1. Introduction

Despite tremendous success of quantum electrodynamics (QED) and unified renormalizable gauge theories [1], calculation of the electron self-energy continues to remain a formidable problem. An ambition of both classical and quantum physics has been to understand all properties of the electron on basis of the electromagnetic force. When Maxwell formulated his theory of electromagnetism to unify optics and electricity, an issue of paramount importance that cropped up as the next step has been explaining the structure of the electron. After Thompson's discovery of the electron and measurement of its charge and mass, the electric charge could not be considered as a fluid. Instead, the electron appeared as the smallest existing unit of the charge as well as the mass. The classical electron theory of Abraham and Lorentz was an ingenious effort in relating electron mass to its charge [2-3]. As a point charge is an obvious impossibility in classical electrodynamics, the electron was endowed with a finite size (radius) determined by equating the electrostatic self-energy to the mass energy Mc^2 of special theory of relativity. Abraham developed an elaborate scheme to derive the inertial mass of the electron as a radiation reaction, however there was a discrepancy of 4/3 in the proportionality factor of between inertial mass and the mass (E/c^2) associated with self-energy E. The other problem of the classical electron was its instability, a consequence of Earnshaws's theorem [4] according to which a system of electric charges cannot be held in equilibrium purely by electromagnetic forces. The above problems of the classical electron were resolved by Poincare [5] who introduced non-electromagnetic cohesive forces (Poincare stresses) to stabilize the electron. Poincare's stress was equivalent to a negative pressure, however Poincare's idea did not receive much attention as it seemed arbitrary and non-electromagnetic forces, apart from weak gravity were not palatable to the physicists at the time.

After advent quantum mechanics, it became clear that classical theory of the electron cannot be correct, because the electron classical radius $r_e = e^2/4\pi\varepsilon_o Mc^2$ is nearly three orders of magnitude smaller than its Compton wavelength $\lambda_e = \hbar/M c$ (i.e. r_e= 2.8 x 10^{-15} m, λ_e = 2.4 x 10^{-12} m). In this situation vacuum polarization needs to be taken into account in any meaningful model of the electron. The first few attempts of constructing models of the electron on quantum mechanical basis were frustrating because the quadratic divergences turned out to be worse than in the classical theory. The quadratic divergence originated from one particle perturbation theory calculation. Weisskopf [6] showed that a calculation based on Dirac's electron-positron theory gives much less serious logarithmic divergence in all orders of the perturbation theory. Subsequent calculations of Feynman [7] , Schwinger [8] and others who used more modern QED, confirmed Weisskopf's result and vacuum polarization contribution δM to the electron mass M was shown to be given by the well known expression,

$$\delta M = \frac{3M\alpha}{2\pi}[\ln(\frac{\Lambda}{M}) + \frac{1}{4}] + \Sigma O(\alpha^n) \tag{1}$$

where α= fine structure constant, Λ= cut-off parameter and O (α^n) denote the n th order correction. The proportionality of the correction to mass of the electron itself guarantees preservation of chiral symmetry in the limit of zero lepton mass [1]. The cut-off parameter is also equivalent to a length R given by $R = (\hbar/\Lambda c)$. However, there was no point electron limit and the problem would not be resolved by full incorporation of higher order terms. The above unsatisfactory feature in QED calculation of electron self-energy appears to be an indication that non-electromagnetic forces are operative in the electron structure at very small scales of length. Most models of electron invoking non-electromagnetic forces attribute a finite size to the electron

[9-11]. As it is natural to believe that the non-electromagnetic force involved is gravitation, many efforts have been made to invoke gravity in regularization of the electron self-energy and analyze gravitating charged spheres [12-20]. A problem of the QED approach to electron self-energy is that unlike in the classical theory, the question of electron stability has been completely overlooked. In the Poincare's classical model of the electron, electromagnetic and non-electromagnetic contributions to self-energy are functions of its radius, minimization of the sum of two contributions gives a unique value for the radius, corresponding to the energy of the stable spherically symmetric configuration.

After discovery of the muon, problem of electron self-energy was further complicated by the existence of another heavier electron like particle. Attempts were made to interpret muon as an excitation of the electron without much success in correctly relating its mass to the electron mass [10]. Following the discovery of the tau-lepton and development of the standard model, the naturalness of the existence of three leptons was evident, however the origin of their masses remain elusive. The Higgs mechanism imply that the existence of charged massive leptons but does not hint a procedure for computation of their masses. It merely implies that charged leptons need to possess a mass proportional to the vacuum expectation value of the Higgs Field.

In this work a semi-classical model of leptons is presented suggesting that the stress of electromagnetic vacuum polarization of an elementary charged object 'pulls out' the true vacuum, to a false vacuum of negative pressure balancing the outward electromagnetic stress. The three lepton flavors corresponds three phases of the false vacuum with different energy densities.

2. Discussion

In the Poincare model of the electron charge Q is considered to be uniformly distributed over a massless spherical shell of radius R. Because of the charge surface density $s = Q/4\pi R^2$, the shell is subject to an outward electrostatic stress $s^2/2\varepsilon_0$, if the interior of sphere is endowed with a negative pressure P, the condition of equilibrium is ,

$$\frac{s^2}{2\varepsilon_0} = P \tag{2}$$

or equivalently,

$$\frac{Q^2}{32\varepsilon_0 \pi^2 r^4} = P \tag{3}$$

The total energy $W(r)$ of the system can be written as,

$$W(r) = \frac{4\pi r^3 P}{3} + \frac{Q^2}{8\varepsilon_0 \pi r} \tag{4}$$

In a modern context, the first term in (4) can be interpreted as the work done against the pressure of the true vacuum (natural vacuum) in creating a false vacuum within the bubble or equivalently the false vacuum possessing a negative pressure. The expression (4) take a minimum value (corresponding to $\mathrm{d}W/\mathrm{d}r = 0$, $\mathrm{d}^2W/\mathrm{d}r^2 > 0$) when the condition (3) is satisfied, giving self-energy of the electron as,

$$W(r) = \frac{4}{3}[\frac{Q^2}{8\varepsilon_0 \pi r}] \tag{5}$$

The quantity under the square bracket in (5) is identical to the second term in (4), the self-energy due to the charge or equivalently the energy of the electric field. Thus total energy of the system is 4/3 times the electrostatic self-energy and if inertial mass is interpreted as W/c^2, the famous 4/3 problem in the Abraham theory is resolved. The theory of Poincare did not receive much attention, because Poincare interpreted $W(r)$ in (3) as electromagnetic energy, although in fact it included a non- electromagnetic contribution of $1/3[Q^2/8\varepsilon_o\pi r]$. Furthermore non-electromagnetic forces apart from weak gravity were foreign to physicists at the time. The Poincare stress tensor is defined as [21],

$$P_{\mu\nu} = -Pg_{\mu\nu} \tag{6}$$

Eqn. (5) was derived considering the rest –frame situation. However, Poincare model incorporating (6) and electromagnetic stress tensor is Lorentz invariant [22]. Eqn. (6) resemble the energy momentum tensor $T_{\mu\nu}$ of a perfect fluid expressed by,

$$T_{\mu\nu} = (\rho + P)U_\mu U_\nu - Pg_{\mu\nu} \tag{7}$$

with $P = -\rho$,where U_μ is the velocity field. Scalar fields also satisfy the condition $P = -\rho$ under certain conditions.

Motivated by the Poincare model of the electron, we consider a spherical bubble of radius R with a constant negative pressure P inside and charge equal to the electronic charge distributed over the surface. The self-energy (mass) M of an elementary object of radius is constituted of two parts ; (i) a non- electromagnetic contribution $(4\pi/3c^2)r^3P$ due to a negative pressure and (ii) the energy due to electromagnetic vacuum polarization in the region out-side the spherical surface given by the QED expression (1) dependent on the cut-off radius $R = \hbar/\Lambda c$. Thus the total energy (mass) of the system is,

$$M = \frac{4\pi R^3 P}{3c^2} + \frac{3M\alpha}{2\pi}[\ln(\frac{\hbar}{McR}) + \frac{1}{4}] \tag{8}$$

Expression (8) is analogous to (4) with the classical electromagnetic energy of the charge distribution replaced by the QED expression for the electron self-energy.

The quantity $M(R)$ given by (9) is extremal ($dM/dr = 0$) when

$$R = [(3\alpha c^2 M)/(8\pi^2 P)]^{1/3} \tag{9}$$

From (8) and (9) it also follows that $d^2M/dr^2 = M/2R^2$ at $dM/dr = 0$ showing that the equilibrium is stable. Eliminating P between (8) and (9), the radius of the bubble can be expressed as,

$$R = (\frac{\hbar}{Mc})\exp(-\frac{2\pi}{3\alpha}+\frac{7}{12}) \tag{10}$$

Similarly eliminating of R between (8) and (9), the stable values of mass M can be written as,

$$M = [\frac{8\pi^2 P\hbar^3}{3\alpha c^5}]^{1/4}\exp(-\frac{\pi}{2\alpha}+\frac{7}{16}) \tag{11}$$

Mass M depends on parameters P and α, the latter should indeed be renormalized value. Energy densities and pressures P are generally expressed in terms of a mass scale μ via the relation,

$$P = \mu^4 c^5 \hbar^{-3} \tag{12}$$

Substituting (12) into (11) we represent lepton masses M_l

$$M_l = [\frac{8\pi^2}{3\alpha}]^{1/4}\mu_l \exp(-\frac{\pi}{2\alpha}+\frac{7}{16}) \tag{13}$$

As there are three leptons l (= e, mu, tau), we need to assume that there are three characteristic mass scales μ_l (equivalently pressures P_l of three phases of the false vacuum) corresponding to each lepton. If the maximum possible mass scale, the Planck scale is assumed to correspond to the tau lepton, then $M_l = M_\tau = 1.777$ GeV/c^2 when $\mu_l = \mu_p$ = Planck mass = 2.435x 10^{18} GeV/c^2. Insertion of these values in (13) and solution of the transcendental equation (13) for α yields $1/\alpha \approx 26.3$. At Planck energies, the fine structure constant would be heavily renormalized and probably there are no constraints to rule out the above value [23-24], which might be infinite energy limit of the fine structure constant. From (13), the scales corresponding to the other two leptons are $\mu_{mu} = 1.45 \times 10^{17}$ and $\mu_e = 6.7 \times 10^{14}$. From (10) it follows that the radii leptons are $R_e = 2.4 \times 10^{-31}$ m, $R_{mu} = 9.3 \times 10^{-33}$ m, $R_{tau} = 2.8 \times 10^{-35}$m. According to model, the tau lepton size is of the order of the Planck length (~ 1.6×10^{-35} m), muon and electron are larger by nearly one and two orders of magnitude respectively. The first term in (1) is non-electromagnetic energy of the lepton. When (9) is substituted this term reduces to $\alpha M/2\pi$. Thus ~ 99.4 % of the self-energy of lepton is electromagnetic in origin. The parameters μ_l could be considered as vacuum expectation values of scalar fields, when this parameter vanishes, lepton masses remain zero.

The radial oscillation angular frequency ω of the bubble in the equilibrium position is $\omega^2 = [c^2 d^2M/dr^2]/M = c^2/2R^2$. Using (10) we obtain,

$$\hbar\omega = Mc^2 Exp (2\pi/3\alpha - 7/12) \tag{14}$$

Thus the excitation energy is of the order of 10^{23} times mass of the respective leptons and practically, leptons have no excited states. Tau and mu are not excited states of the electron.

The existence of three leptons with same charge but different masses suggests that the mass is determined by another parameter in addition to the charge. In terms of the present model, the other parameter is value of negative pressure inside the bubble defined in terms μ_l by (13). Thus flavor corresponds to existence of the phases of the false vacuum, suggesting that flavor is an attribute of the vacuum structure.

3. Conclusion

The idea presented here is analogous to the Poincare model of the electron, where the negative pressure inside a charged bubble is balanced by an outward stress. However, unlike in the latter case the outward stress originate from vacuum polarization. Energy of the entity is constituted of two terms. The energy due to negative pressure (i.e. positive energy density) proportional to the volume of the bubble and energy of vacuum polarization taken to be the QED self-energy expression. Minimization of total energy yields the equilibrium condition. Lepton masses depend on renormalized fine structure constant and a mass scale defining the negative pressure. Existence of three leptons with different flavors is attributed to existence of three phases of the false vacuum corresponding to different negative pressures. The assumption that the negative pressure associated with the heaviest lepton (tau) is Planck pressure yields the renormalized value of the fine structure constant at extreme energies. The radii of the leptons are evaluated. Tau lepton size is of the order of the Planck length (~ 1.6 x 10^{-35} m), muon and electron are larger by nearly one and two orders of magnitude respectively. Thus leptons are effectively point like particles and the model does not contradict QED. An experimental upper limit for the electron radius is of the order 10^{-23}m [25].The model assumes that the electron structure possess spherical symmetry. A recent experiment indicates that vacuum polarization field around the electron is indeed spherically symmetrical and the electron carries no dipole moment [26].

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