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Effect of streaming and temperature anisotropy in Wiedemann-Franz law for collisional magnetoplasma

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Abstract

The presence of beam in a two component fully ionized electron-ion magneto plasma entails streaming or mass motion of plasma species causing $\vec{V}_0.\vec{P}$ energy dissipation where $\vec{V}_0$ represents streaming velocity. The collisional diffusion transport coefficients are significantly modified in presence of the streaming velocity $\vec{V}_0$. Further these coefficients are also found to be modified owing to anisotropy in plasma species temperature. The modified transport coefficients results in significant modifications of Wiedemann-Franz ratio (k/σ) in terms of $\vec{V}_0$ and $T_{\parallel}/T_{\perp}$, where $T_{\parallel}$ and $T_{\perp}$ represent the temperature along and perpendicular to the direction of $\vec{B}$ field.

Keywords: *Wiedemann-Franz Ratio, Streaming, Non-Streaming, Anisotropy*

1. Introduction

Recent advances [1-3] in the study of collisional diffusion and transport in a two component fully ionized (electro-ion) magneto plasma having anisotropies in temperature ($T_{\parallel}$and $T_{\perp}$) and streaming velocity ($\vec{V}_0$) has aroused current interest in studying the Wiedemann-Franz law for gaseous plasma.

Wiedemann and Franz (1853) gave the empirical law that ratio of the thermal and electrical conductivities (k/σ) at a particular temperature is same for all metals. Lorentz (1872) showed that the ratio K/σ is proportional to the absolute temperature. Experimentally it is observed that the law holds good at ordinary temperature but fails at low temperature owing to the contributions due to phonons. Paul Drude (1900) proposed the free electron model and explained Wiedemann and Franz law assuming that the free electrons undergoing collisions with atoms of the metals are in thermal equilibrium with Maxwell – Boltzmann velocity distribution and calculated that $\frac{K}{\sigma T} = \frac{3k^2}{2e^2} = 1.11\times10^{-8}$ W. Ω /k^2, although the corrected value is calculated as $\frac{K}{\sigma T} = \frac{\pi^2 h^2}{3e^2}$ =2.45×10^{-10} W. Ω /k^2 based on Fermi Dirac statistics.

In the present paper we attempt to study the behavior of the Wiedemann-Franz ratio (K/σ) for a fully ionized singly charged electron-ion magneto plasma although unlike the metals, an increase in the electrical conductivity causes a decrease in thermal conductivity in a plasma. We further study the dependence of this ratio on the streaming anisotropy [2] as well as the anisotropy in plasma temperature [3] and compare the results with the isotropic ones [4] both quantitatively and qualitatively.

2. Basic Theory and Derivations

It is well known that fully ionized plasma gets diffused across a magnetic field by means of interparticle collisions when a transverse density gradient exists [5]. The presence of a temperature gradient, in addition to the density gradient in the same direction causes the thermal energy transport along the cross field direction. The relevant Boltzmann transport equation is solved by Rosenbluth and Kaufmann [4] to yield expressions for the electrical resistivity ($\eta_\perp$) and the thermal conductivity K as,

$$\eta_\perp = \frac{8}{3}\left(\frac{\pi}{2}\right)^{1/2} e^2 \frac{(m^-)^{1/2}}{(kT)^{3/2}} \log\left(\frac{2}{\theta_0}\right) \tag{1}$$

and

$$K = \frac{1}{4}\left(\frac{2m^+}{m^-}\right)^{1/2} \frac{N^2C^2}{B^2} kT\,\eta_\perp \tag{2}$$

Where N is density, C is speed of light and θ_0 is minimum angle of scattering [2].

If the ion mass (m_i) > > the electron mass (m_e), then ratio of the thermal conductivity to the electrical conductivity yields

$$\frac{K}{\sigma_\perp} = \frac{K}{1/\eta_\perp} = K\eta_\perp = \frac{8}{9}\pi\left(2m^+m^-\right)^{1/2} \frac{N^2C^2}{B^2} \frac{e^2}{(kT)^2} \log\left(\frac{2}{\theta_0}\right) \tag{3}$$

where the Coulomb logarithm takes appropriate values for weakly coupled plasma [3]. It is worthwhile to note that the Wiedemann-Franz ratio $K/\sigma_\perp$ is inversely proportional to $(kT)^2$ for a gaseous plasma where as for metals it is found to be directly proportional to the thermal energy kT or the absolute temperature T. However, for constant temperature, $\frac{K}{\sigma}$ remains constant although it is different from that of the metal owing to the appearance of $(kT)^2$ term in the denominator and the squared density term (N^2) in the numerator. Note that for metals $K/\sigma_\perp$ is independent of the density term. However, on keeping the values of N and the magnetic field $\vec{B}$ constant at a constant temperature, the ratio $K/\sigma_\perp$ assumes a constant value. However, this value decreases rapidly with increase in kT.

Mohanty and Baral [2] solved the Boltzmann transport equation with an equilibrium distribution function for a plasma having anisotropy is streaming velocity $\vec{V}_o$.

$$f_o^j = \frac{N}{2}\left(\frac{m^j}{2\pi kT_j}\right)^{3/2} \exp\left(-\frac{m^j V_0^2}{2kT_j}\right) \exp\left[-\frac{1}{kT_j}\left(\frac{p_j^2}{2m^j} - \vec{V}_0 \cdot \vec{p}_j\right)\right] \tag{4}$$

It is revealed that the electrical and thermal conductivities are modified as,

$$\eta_\perp = \frac{8}{3}\left(\frac{\pi}{2}\right)^{1/2} \frac{(m^-)^{1/2}}{(kT)^{3/2}} e^2 \log\left(\frac{2}{\theta_0}\right) \tag{5}$$

and

$$K = 10\left(1 - \frac{7}{24}\frac{m^+ V_0^2}{kT}\right)\left(\frac{2m^+}{m^-}\right)^{1/2} \frac{N^2 C^2}{B^2} kT \eta_\perp \tag{6}$$

On simplification the Wiedemann- Franz ratio is derived as

$$\left(\frac{k}{\sigma_\perp}\right)_{streaming} = 80\pi\left(\frac{1 - \frac{2}{3}\frac{m^+ V_0^2}{kT}}{1 - \frac{m^- V_0^2}{kT}}\right)\left(2m^+ m^-\right)^{1/2} \frac{N^2 C^2}{B^2} \frac{e^4}{(kT)^2}\left\{\log\left(\frac{2}{\theta_0}\right)\right\}^2 \tag{7}$$

The comparison reveals that the ratio $\left(\frac{K}{\sigma_\perp}\right)$ increases to $90\left(\frac{1 - \frac{2}{3}\frac{m^+ V_0^2}{kT}}{1 - \frac{m^- V_0^2}{kT}}\right)$ times the ratio for non-streaming plasma. Note that in our streaming model, the streaming energy $m^j V_0^2 < kT_j$ i.e. $V_0^2 < U_{th}^2$.

Likewise in our early work [3] we have attempted to derive the expressions for modified diffusion transport coefficients in a fully ionized singly charged two component magnetoplasma at the onset of an anisotropy in plasma temperature $(T_\parallel \neq T_\perp)$, where $T_\parallel$ and $T_\perp$ represent the temperature along and perpendicular to the direction of $\vec{B}$ field. The relevant Maxwellian equilibrium distribution function is,

$$f_0^j = \frac{N}{2}\left(\frac{m^j}{2\pi kT_\parallel}\right)\left(\frac{m^j}{2\pi kT_\parallel}\right)^{1/2} \exp\left[-\left\{\frac{m^j\left(V_x^2 + V_y^2\right)}{2kT_\perp}\right\} + \frac{m^j V_z^2}{2kT_\parallel}\right] \tag{8}$$

On solving Boltzmann-transport equation in a Chapman-Enskog approximation method following our early work [3], it is further revealed that both the conductivities are functions of the temperature ratio $\left(\frac{T_\parallel}{T_\perp}\right)$ as

$$\eta_\perp = \frac{8}{3}\left(\frac{\pi}{2}\right)^{1/2}\left[\frac{27T_\parallel/T_\perp}{2\frac{T_\parallel}{T_\perp} + 1}\right]^{1/2} \frac{(m^-)^{1/2}}{(kT_\perp)^{3/2}(T_\parallel/T_\perp)^{1/2}} e^2 \log\left(\frac{2}{\theta_0}\right) \tag{9}$$

and

$$K = \frac{2}{3}\left(11 + \frac{T_\parallel}{T_\perp}\right)\left(\frac{2m^+}{m^-}\right)^{1/2} \frac{N^2 C^2}{B^2} e^4 kT_\perp \eta_\perp \tag{10}$$

The ratio of the conductivities is calculated as

$$\frac{K}{\sigma_\perp} = 576\pi(11+\frac{T_{||}}{T_\perp})\frac{1}{(kT)\left(2\frac{T_{||}}{T_\perp}+1\right)}(2m^+m^-)^{\frac{1}{2}}\frac{N^2C^2}{B^2}e^4\left\{\log\left(\frac{2}{\theta_0}\right)\right\}^2 \tag{11}$$

This equation involves $T_{||}/T_\perp$ dependence of the Wiedemann-Franz ratio.

3. Discussion

The dependence of the Wiedemann-Franz ration ($K/\sigma_\perp$) on the temperature for an isotropic two component electron-ion magneto plasma is derived in Eq. (3). The Results are quantitatively estimated for various temperatures and are displayed in Fig.1. It shows that ratio almost remains constant within the range from 100ev –800ev. It is instructive to note that in contradistinction to the Wiedemann-Franz law for metallic conductors, the gaseous plasma shows a decrease in electrical conductivity, the gaseous plasma shows a decrease in electrical conductivity with increase in thermal conductivity in the lower thermal regimes and remains dependant on the plasma species density. However, it remains constant for constant temperature at particular value of the magnetic field $\vec{B}$ and the plasma species density (N).

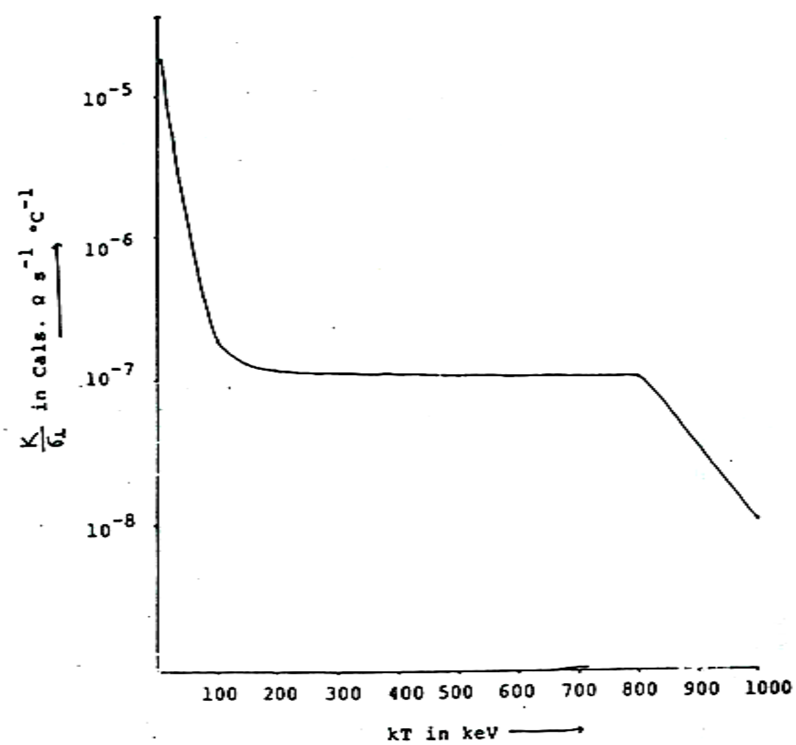

Fig.1. Variation of ($K/\sigma_\perp$) with thermal energy (kT) for isotropic electron ion magneto-plasma.

Towards higher thermal regimes $K/\sigma_\perp$ drops because of large decrease of the thermal conductivity (K). In Fig. 2 and Fig. 3 qualitative results are plotted to show that the ratios are dependent on the streaming velocity ($\vec{V}_0$) while Fig. 4 shows the variations of $K/\sigma_\perp$ for various temperature ratios $T_{||}$ / $T_\perp$. The results are of importance for study the electrical and thermal properties of plasma relevant to Laboratory plasma and plasmas in space and astrophysical situations.

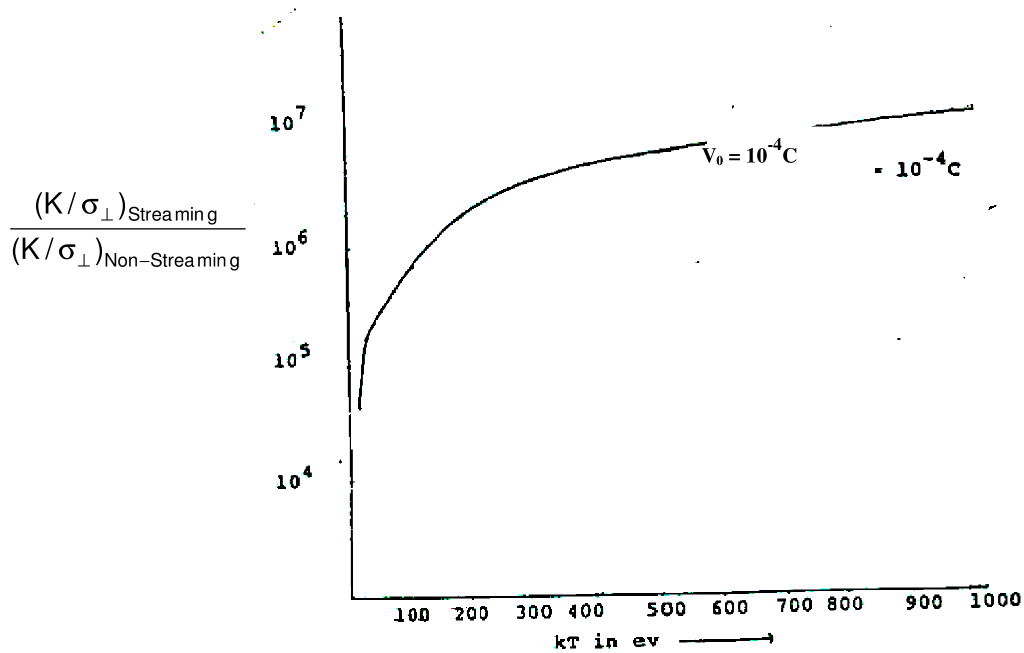

Fig.2. Variation of $\frac{(K/\sigma_{\perp})_{Streaming}}{(K/\sigma_{\perp})_{Non-Streaming}}$ with thermal energy (kT)

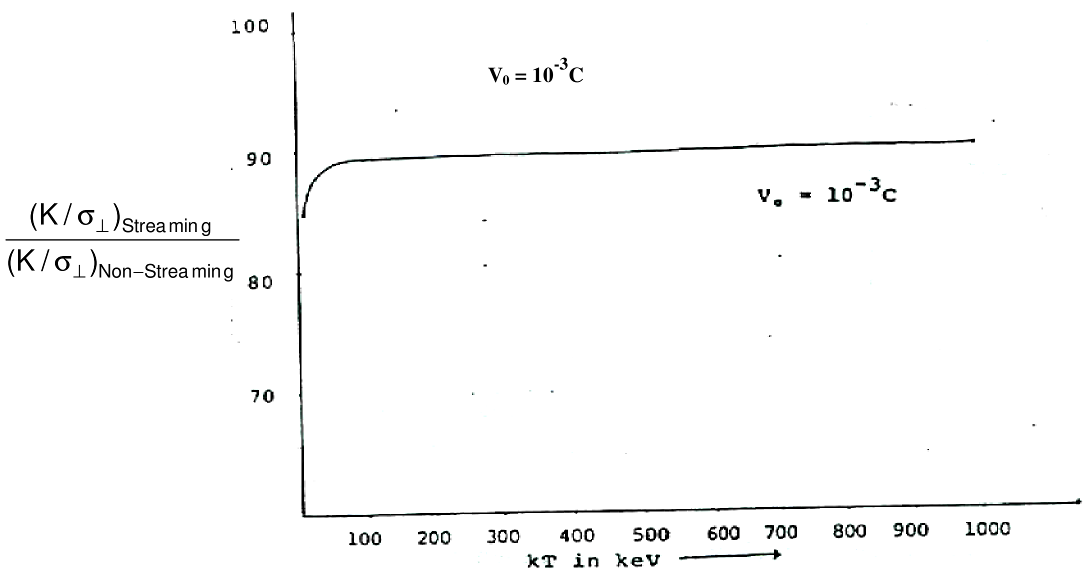

Fig.3. Variation of $\frac{(K/\sigma_{\perp})_{Streaming}}{(K/\sigma_{\perp})_{Non-Streaming}}$ with thermal energy (kT)

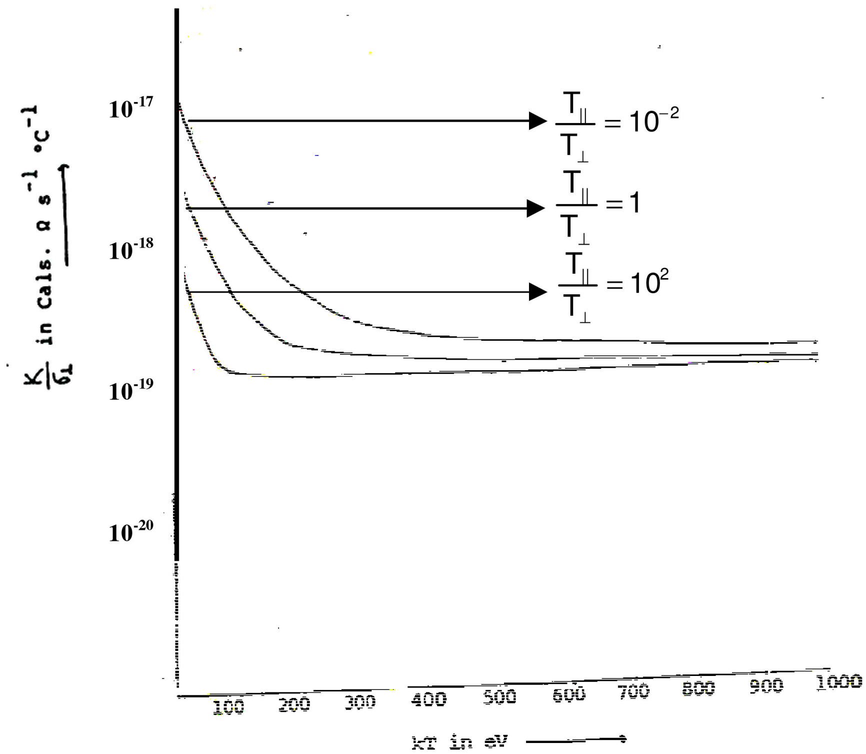

Fig. 4 Variation of $(K/\sigma_\perp)$ with thermal energy $(kT_\perp)$ for various temperature ratios $\left(\frac{T_\parallel}{T_\perp}\right)$

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REFERENCES

[1] J N Mohanty, K.C. Baral, "Cross field diffusion and transport in a nonrelativistic collisional kinetics", *Phys. Plasma,* **3**(12), 4410-4414 (1996)

[2] J N Mohanty, K.C. Baral, "Cross field diffusion and transport in a nonrelativistic beam-driven magnetoplasma: Collisional kinetics", *Phys. Plasma*, **8**(12), 5134-5139 (2001)

[3] K C Baral and G. Nath, "Diffusion and transport phenomena in a collisional magnetoplasma with temperature anisotropy", *Indian Journal of Pure & Applied Physics,* **46**, 111-117 (2008)

[4] M N Rosenbluth and A.N. Kaufman, "Plasma Diffusion in a Magnetic Field", *Phys, Rev.*, **109**, 1-5(1958)

[5] C Longmire and M. Rosenbluth, "Diffusion of Charged Particles across a Magnetic Field", *Phys. Rev,* **103**, 507-510 (1956)