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Comment on Aspect's experiment: classical interpretation

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Abstract

Quantum mechanics was the foundation for physics in the 20th century and its mysterious world has presented various unique effects beyond human understanding. In particular, Aspect's experiment and Bell's inequality suggest a non-local interaction causing wave packet reduction, and are regarded as evidence for quantum mechanics' validity. This short paper reconsiders the electric field of entangled light and Aspect's experiment in terms of classical theory and shows that the experimental results can be explained equally as well.

Keywords**:** Entangled state, Aspect's experiment, Simultaneous measurement, Classical theory

1. Introduction

Quantum mechanics has correctly explained phenomena that are difficult to understand intuitively and has been applied to many research fields and industries. The theoretical basis for quantum mechanics are duality [1] and wave packet reduction [2], but these are hard to interpret, and remain active and attractive research topics.

We have previously shown that interference fringes and photon trajectories can be measured simultaneously using a modified Young's double slit, which questions the duality assumption [3]; and we have also shown experimentally that the wave packet does not collapse using fourth order interference [4]. Although we were conducting experiments to verify the theoretical basis for quantum mechanics, these two results were contrary to the underlying assumptions.

This paper reconsiders the classical theory interpretation for optical Bell test experiments, such as Aspect's experiments [5–7]. Entangled photon pairs, where the polarization direction of the photon pair emitted in each direction is parallel or perpendicular, are commonly used to verify Bell's inequality experimentally. Several light sources generate entangled photon pairs, but for simplicity we consider spontaneous parametric down-conversion (SPDC) as the light source [8]. Incorporating entangled photon pair conditions into the electromagnetic field and classical theory, we show that the simultaneous measurement probability is the same as the result from quantum mechanics.

2. Aspect's experiment and entangled state

Figure 1 outlines Aspect's experimental equipment. An entangled photon pair is emitted from the light source and travels to the left and right. Photons traveling to the left pass through polarizer A (θ_1 from the horizontal direction), whereas photons traveling to the right pass through polarizer B (θ_2 from the horizontal direction) and are simultaneously measured.

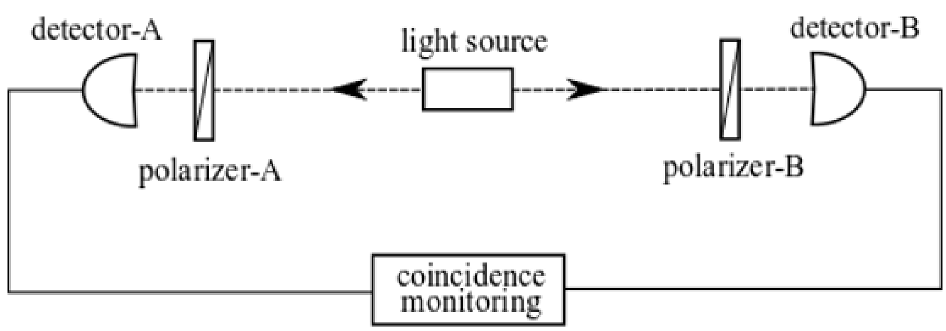

Fig. 1. Aspect's experimental apparatus. Entangled photon pairs are emitted to the left and right. Each photon passes through polarizing plates A and B (polarization directions θ_1 and θ_2), and the probability of simultaneous detection is measured.

Consider the two photon entangled state ("EPR-Bell states" [9]),

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_A |H\rangle_B + |V\rangle_A |V\rangle_B\right) \tag{1}$$

$$|\psi'\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_A |V\rangle_B + |V\rangle_A |H\rangle_B\right) \tag{1'}$$

where H and V represent polarized light in the horizontal and vertical direction, respectively; and A and B represent the detector direction. The relative phase shift between the first and second terms is omitted since it can be set to any desired value, e.g. zero, using a phase shifter [8]. In the photon number state, Eqs. (1) and (1') can be expressed as

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|1,0,1,0\rangle + |01,0,1\rangle\right) \tag{2}$$

$$|\psi'\rangle = \frac{1}{\sqrt{2}}\left(|1,0,0,1\rangle + |01,1,0\rangle\right) \tag{2'}$$

where |n1,n2,n3,n4> represents the number of horizontally polarized photons in the A direction, the number of vertically polarized photons in the A direction, the number of horizontally polarized photons in the B direction, and the number of vertically polarized photons in the B direction, respectively. The positive frequency part of the fields at detectors A and B are

$$\begin{aligned} \hat{E}_A^{(+)}(\boldsymbol{r}_A) &= (\hat{a}_1 \cos\theta_1 + \hat{a}_2 \sin\theta_1) e^{i\boldsymbol{k}_A \cdot \boldsymbol{r}_A} \\ \hat{E}_B^{(+)}(\boldsymbol{r}_B) &= (\hat{a}_3 \cos\theta_2 + \hat{a}_4 \sin\theta_2) e^{i\boldsymbol{k}_B \cdot \boldsymbol{r}_B} \end{aligned} \tag{3}$$

where $\boldsymbol{k}_A$ and $\boldsymbol{k}_B$ are wave vectors, and $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$ are position vectors of the detector; and $\hat{a}_1$, $\hat{a}_2$, $\hat{a}_3$ and $\hat{a}_4$ are photon annihilation operators. Simultaneous photon measurement probabilities for Eqs. (2) and (2') are

$$\begin{aligned} P_q(\boldsymbol{r}_A, \boldsymbol{r}_B) &= \left\langle \hat{E}_A^{(-)}(\boldsymbol{r}_A) \hat{E}_B^{(-)}(\boldsymbol{r}_B) \hat{E}_B^{(+)}(\boldsymbol{r}_B) \hat{E}_A^{(+)}(\boldsymbol{r}_A) \right\rangle \\ &= \frac{1}{2}\cos^2(\theta_1 - \theta_2) \end{aligned} \tag{4}$$

$$P_q'(\boldsymbol{r}_A, \boldsymbol{r}_B) = \frac{1}{2}\sin^2(\theta_1 - \theta_2) \tag{4'}$$

Thus, simultaneous photon detection probability is proportional to $\cos^2(\theta_1\text{-}\theta_2)$ or $\sin^2(\theta_1\text{-}\theta_2)$, which has maximum or minimum at $\theta_1 = \theta_2$ [9]. The form of these expressions is the cause for contrary to Bell’s inequality [5–7] and at the moment the photon is detected by detector A, it seems that the photon on the B side collapses into a polarization state parallel (or perpendicular) to θ_1, suggesting a "spooky " non-local interaction, i.e., wave packet reduction.

If we replace the operator in Eq. (3) with the classical number (c-number), we obtain the result of classical theory and find that the result satisfies Bell's inequality. However, the entangled states of Eqs. (1) and (1 ') can be considered as different light sources, but in classical theory they are given by the same electric field (assuming that $|a_1| = |a_2| = |a_3| = |a_4|$). It is necessary to re-examine the form of Eq. (3).

3. Aspect's experiment interpreted by classical theory

Let us consider the generation process of the entangled state of Eq. (1) with the positive frequency part of the electric field of the excitation light as exp $[i(\boldsymbol{k}\cdot\boldsymbol{r} + \omega t)]$. Since the electric field generated from the nonlinear crystal simultaneously contains the polarized components in the horizontal and vertical directions, the excitation light is divided into two electric fields. Each electric field is further separated into two electric fields (photons) while preserving the momentum and energy, and the entangled light is generated. This process can be expressed by the following equation.

$$\begin{aligned} E^{(+)}(\boldsymbol{r},t) &= A_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{r}+\omega t)} \\ &\propto A_1 e^{i(\boldsymbol{k}\cdot\boldsymbol{r}+\omega t)} + A_2 e^{i(\boldsymbol{k}\cdot\boldsymbol{r}+\omega t)} \\ &= A_1 e^{i(\boldsymbol{k}_1\cdot\boldsymbol{r}+\omega_1 t)} e^{i(\boldsymbol{k}_2\cdot\boldsymbol{r}+\omega_2 t)} + A_2 e^{i(\boldsymbol{k}_3\cdot\boldsymbol{r}+\omega_3 t)} e^{i(\boldsymbol{k}_4\cdot\boldsymbol{r}+\omega_4 t)} \end{aligned} \tag{5}$$

Here, $\boldsymbol{k}$, $\boldsymbol{k}_1$, $\boldsymbol{k}_2$, $\boldsymbol{k}_3$ and $\boldsymbol{k}_4$ are wave vectors, $\boldsymbol{r}$ is the position vector, ω, ω_1, ω_2, ω_3 and ω_4 are angular frequencies, and A_0, A_1 and A_2 are the amplitudes of the electric field. To be consistent with Eq. (1), i.e., the tensor-product state, the entangled light can be expressed in terms of the product, not sum, of the electric field as shown in Eq. (5). The first term consists of two electric fields of polarized light in the horizontal direction (H), and the second term consists of two vertically polarized electric fields (V). When we express the electric field as shown in Eq. (5), measuring the first (or second) term means to perform a "simultaneous measurement" of the entangled light (or photon pairs) that are polarized in the horizontal (or vertical) direction and also means measuring the excitation light (single photon).

By transforming Eq. (5), we obtain the electric field in the experiment of Aspect in Fig. 1 as follows (assuming that A_1=A_2=A, $\boldsymbol{k}_1$=$\boldsymbol{k}_3$=$\boldsymbol{k}_\mathrm{A}$ and $\boldsymbol{k}_2$=$\boldsymbol{k}_4$=$\boldsymbol{k}_\mathrm{B}$).

$$\begin{aligned} E^{(+)}(\boldsymbol{r}_A,\boldsymbol{r}_B,t) &= A\left(\cos\theta_1 e^{i(\boldsymbol{k}_A\cdot\boldsymbol{r}_A+\omega_1 t)}\cos\theta_2 e^{i(\boldsymbol{k}_B\cdot\boldsymbol{r}_B+\omega_2 t)} + \sin\theta_1 e^{i(\boldsymbol{k}_A\cdot\boldsymbol{r}_A+\omega_3 t)}\sin\theta_2 e^{i(\boldsymbol{k}_B\cdot\boldsymbol{r}_B+\omega_4 t)}\right) \\ &= A\left(\cos\theta_1 e^{i\boldsymbol{k}_A\cdot\boldsymbol{r}_A}\cos\theta_2 e^{i\boldsymbol{k}_B\cdot\boldsymbol{r}_B} + \sin\theta_1 e^{i\boldsymbol{k}_A\cdot\boldsymbol{r}_A}\sin\theta_2 e^{i\boldsymbol{k}_B\cdot\boldsymbol{r}_B}\right)e^{i\omega t} \end{aligned} \tag{6}$$

The probability of simultaneous measurement is

$$
\begin{aligned}
P_c(r_A, r_B) &= \left| E^{(+)}(\boldsymbol{r}_A, \boldsymbol{r}_B, t) \right|^2 \\
&= A^2 \left(\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \right)^2 \\
&= A^2 \cos^2(\theta_1 - \theta_2) \quad . \qquad (7)
\end{aligned}
$$

Thus, the probability of simultaneous measurement is given by the "second-order" interference. Since A^2 = 1/2 by the normalization of Eq. (6), it is identical to Eq. (4) obtained by quantum mechanics.

In addition, the expression by the quantum-number (q-number) in Eq. (5) is

$$
\widehat{E}^{(+)}(\boldsymbol{r_A}, \boldsymbol{r_B}, t) = \left(\hat{a}_1 \cos\theta_1 e^{i\boldsymbol{k}_A \cdot \boldsymbol{r}_A} \hat{a}_3 \cos\theta_2 e^{i\boldsymbol{k}_B \cdot \boldsymbol{r}_B} + \hat{a}_2 \sin\theta_1 e^{i\boldsymbol{k}_A \cdot \boldsymbol{r}_A} \hat{a}_4 \sin\theta_2 e^{i\boldsymbol{k}_B \cdot \boldsymbol{r}_B} \right) e^{i\omega t} \; . \qquad (8)
$$

Using Eq. (2), the probability of simultaneous measurement by quantum mechanics is

$$
\begin{aligned}
P_{q2}(\boldsymbol{r}_A, \boldsymbol{r}_B, t) &= \left\langle \hat{E}^{(-)}(\boldsymbol{r}_A, \boldsymbol{r}_B, t) \hat{E}^{(+)}(\boldsymbol{r}_A, \boldsymbol{r}_B, t) \right\rangle \\
&= \frac{1}{2} \cos^2(\theta_1 - \theta_2) \, . \qquad (9)
\end{aligned}
$$

Even in the electric field expressed by Eq. (5), the calculation of quantum mechanics can be correctly performed. Equations (4') is also obtained using similar calculations on the states of Eqs. (1') and (2').

In addition, the fourth-order interference with 100% visibility (quantum mechanical effect) indicated by Mandel et al. [10-12] can be obtained with 100% visibility in classical theory or quantum theory using Eq. (5). We are currently considering applying this method to other simultaneous measurements. Thus, by representing the entangled light by the product of the electric field, we represent the simultaneous measurement by the second-order interference, and quantum mechanics, classical theory and experiment have consistent results with one another. Therefore, the discussion of the "spooky" non-local correlation that was suggested by Aspect's experiment is unnecessary and cannot be used as experimental evidence of the wave packet reduction.

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